

Mathematics

Advanced GCE **A2 7890 – 2**

Advanced Subsidiary GCE **AS 3890 – 2**

Mark Schemes for the Units

June 2009

3890-2/7890-2/MS/R/09

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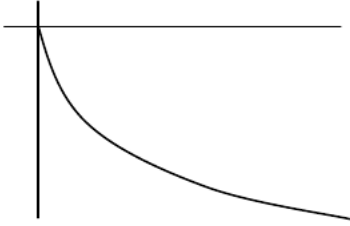
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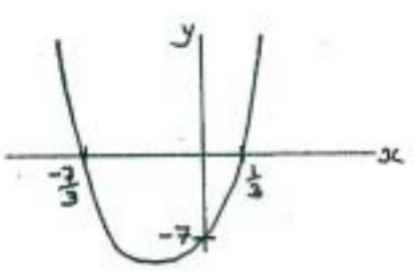
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4721 Core Mathematics 1

1 (i)	$\frac{dy}{dx} = 5x^4 - 2x^{-3}$	B1 M1 A1 3	$5x^4$ x^{-2} before differentiation or kx^{-3} in $\frac{dy}{dx}$ soi $-2x^{-3}$
(ii)	$\frac{d^2y}{dx^2} = 20x^3 + 6x^{-4}$	M1 A1 2 5	Attempt to differentiate their (i) – at least one term correct cao
2	$\frac{(8 + \sqrt{7})(2 - \sqrt{7})}{(2 + \sqrt{7})(2 - \sqrt{7})}$ $= \frac{9 - 6\sqrt{7}}{4 - 7}$ $= -3 + 2\sqrt{7}$	M1 A1 A1 A1 4 4	Multiply numerator and denominator by conjugate Numerator correct and simplified Denominator correct and simplified cao
3 (i)	3^{-2}	B1 1	
(ii)	$3^{\frac{1}{3}}$	B1 1	
(iii)	$3^{10} \times 3^{30}$ $= 3^{40}$	M1 A1 2 4	3^{30} or 9^{20} soi
4	$y = 2x - 4$ $4x^2 + (2x - 4)^2 = 10$ $8x^2 - 16x + 16 = 10$ $8x^2 - 16x + 6 = 0$ $4x^2 - 8x + 3 = 0$ $(2x - 1)(2x - 3) = 0$ $x = \frac{1}{2}, x = \frac{3}{2}$ $y = -3, y = -1$	M1* A1 M1dep* A1 A1 A1 6 6	Attempt to get an equation in 1 variable only Obtain correct 3 term quadratic (aef) Correct method to solve quadratic of form $ax^2 + bx + c = 0$ ($b \neq 0$) Correct factorisation oe Both x values correct Both y values correct <u>or</u> one correct pair of values www B1 second correct pair of values B1

<p>5 (i)</p> $(2x^2 - 5x - 3)(x + 4)$ $= 2x^3 + 8x^2 - 5x^2 - 20x - 3x - 12$ $= 2x^3 + 3x^2 - 23x - 12$ <p>(ii)</p> $2x^4 + 7x^4$ $= 9x^4$ <p>9</p>		<p>M1</p> <p>A1</p> <p>A1 3</p> <p>B1</p> <p>B1 2</p> <p>5</p>	<p>Attempt to multiply a quadratic by a linear factor or to expand all 3 brackets with an appropriate number of terms (including an x^3 term)</p> <p>Expansion with no more than one incorrect term</p> <p>$2x^4$ or $7x^4$ soi www</p> <p>$9x^4$ or 9</p>
<p>6 (i)</p>  <p>(ii)</p> <p>Translation Parallel to y-axis, 5 units</p> <p>(iii)</p> $y = -\sqrt{\frac{x}{2}}$		<p>B1</p> <p>B1 2</p> <p>B1</p> <p>B1 2</p> <p>M1</p> <p>A1 2</p> <p>6</p>	<p>One to one graph <u>only</u> in bottom right hand quadrant</p> <p>Correct graph, passing through origin</p> <p>$\sqrt{2x}$ or $\sqrt{\frac{x}{2}}$ seen</p> <p>cao</p>
<p>7 (i)</p> $\left(x - \frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + \frac{1}{4}$ $= \left(x - \frac{5}{2}\right)^2 - 6$ <p>(ii)</p> $\left(x - \frac{5}{2}\right)^2 - 6 + y^2 = 0$ <p>Centre $\left(\frac{5}{2}, 0\right)$</p> <p>Radius = $\sqrt{6}$</p>		<p>B1</p> <p>M1</p> <p>A1 3</p> <p>B1</p> <p>B1</p> <p>B1 3</p> <p>6</p>	<p>$a = \frac{5}{2}$</p> <p>$\frac{1}{4} - a^2$</p> <p>cao</p> <p>Correct x coordinate</p> <p>Correct y coordinate</p>

<p>8 (i)</p> $-42 < 6x < -6$ $-7 < x < -1$ <p>(ii)</p> $x^2 > 16$ $x > 4$ <p>or</p> $x < -4$		<p>M1</p> <p>A1</p> <p>A1 3</p> <p>B1</p> <p>B1</p> <p>B1 3</p> <p>6</p>	<p>2 equations or inequalities both dealing with all 3 terms</p> <p>-7 and -1 seen oe</p> <p>$-7 < x < -1$ (or $x > -7$ and $x < -1$)</p> <p>± 4 oe seen</p> <p>$x > 4$</p> <p>$x < -4$ not wrapped, not 'and'</p>
<p>9 (i)</p> $\sqrt{(-1-4)^2 + (9-3)^2}$ <p>=13</p> <p>(ii)</p> $\left(\frac{4+1}{2}, \frac{-3+9}{2}\right)$ $\left(\frac{3}{2}, 3\right)$ <p>(iii)</p> <p>Gradient of $AB = -\frac{12}{5}$</p> $y - 3 = -\frac{12}{5}(x - 1)$ $12x + 5y - 27 = 0$		<p>M1</p> <p>A1 2</p> <p>M1</p> <p>A1 2</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1 4</p> <p>8</p>	<p>Correct method to find line length using Pythagoras' theorem</p> <p>cao</p> <p>Correct method to find midpoint</p> <p>Correct equation for line, any gradient, through (1, 3)</p> <p>Correct equation in any form with gradient simplified</p> <p>$12x + 5y - 27 = 0$</p>
<p>10 (i)</p> $(3x + 7)(3x - 1) = 0$ $x = -\frac{7}{3}, x = \frac{1}{3}$ <p>(ii)</p> $\frac{dy}{dx} = 18x + 18$ $18x + 18 = 0$ $x = -1$ $y = -16$ <p>(iii)</p>  <p>(iv)</p> $x > -1$		<p>M1</p> <p>A1</p> <p>A1 3</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1 ft 4</p> <p>B1</p> <p>B1</p> <p>B1 3</p> <p>B1 1</p> <p>11</p>	<p>Correct method to find roots</p> <p>Correct factorisation oe</p> <p>Correct roots</p> <p>Attempt to differentiate y</p> <p>Uses $\frac{dy}{dx} = 0$</p> <p>Positive quadratic curve</p> <p>y intercept (0, -7)</p> <p>Good graph, with correct roots indicated and minimum point in correct quadrant</p>

<p>11 (i)</p>	<p>Gradient of normal = $-\frac{2}{3}$</p> $\frac{dy}{dx} = \frac{1}{2}kx^{\frac{1}{2}}$ <p>When $x = 4$, $\frac{dy}{dx} = \frac{k}{4}$</p> $\therefore \frac{k}{4} = \frac{3}{2}$ $k = 6$	<p>B1</p> <p>M1*</p> <p>A1</p> <p>M1dep*</p> <p>M1dep*</p> <p>A1 6</p>	<p>Attempt to differentiate equation of curve</p> $\frac{1}{2}kx^{-\frac{1}{2}}$ <p>Attempt to substitute $x = 4$ into their $\frac{dy}{dx}$ so i</p> <p>Equate their gradient expression to negative reciprocal of their gradient of normal</p> <p>cao</p>
<p>(ii)</p>	<p>P is point (4, 12)</p> <p>Q is point (22, 0)</p> $\text{Area of triangle} = \frac{1}{2} \times 12 \times 22$ $= 132 \text{ sq. units}$	<p>B1 ft</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1 5</p> <p>11</p>	<p>Correct method to find coordinates of Q</p> <p>Correct x coordinate</p> <p>Must use y coordinate of P and x coordinate of Q</p>

4722 Core Mathematics 2

- 1 (i) $\cos \theta = \frac{6.4^2 + 7.0^2 - 11.3^2}{2 \times 6.4 \times 7.0}$ M1 Attempt use of cosine rule (any angle)
 $= -0.4211$ A1 Obtain one of 115° , 34.2° , 30.9° , 2.01 , 0.597 , 0.539
 $\theta = 115^\circ$ or 2.01 rads A1 3 Obtain 115° or 2.01 rads, or better

- (ii) area $= \frac{1}{2} \times 7 \times 6.4 \times \sin 115$ M1 Attempt triangle area using $(\frac{1}{2})ab \sin C$, or equiv
 $= 20.3 \text{ cm}^2$ A1 2 Obtain 20.3 (cao)

5

- 2 (i) $a + 9d = 2(a + 3d)$ M1* Attempt use of $a + (n - 1)d$ or $a + nd$ at least once for u_4 ,
 u_{10} or u_{20}
 $a = 3d$ A1 Obtain $a = 3d$ (or unsimplified equiv) and $a + 19d = 44$
 $a + 19d = 44 \Rightarrow 22d = 44$ M1dep* Attempt to eliminate one variable from two simultaneous
equations in a and d , from u_4 , u_{10} , u_{20} and no others
 $d = 2$, $a = 6$ A1 4 Obtain $d = 2$, $a = 6$

- (ii) $S_{50} = \frac{50}{2} (2 \times 6 + 49 \times 2)$ M1 Attempt S_{50} of AP, using correct formula, with $n = 50$,
allow $25(2a + 24d)$
 $= 2750$ A1 2 Obtain 2750

6

- 3 $\log 7^x = \log 2^{x+1}$ M1 Introduce logarithms throughout, or equiv with base 7 or 2
 $x \log 7 = (x + 1) \log 2$ M1 Drop power on at least one side
 $x(\log 7 - \log 2) = \log 2$ A1 Obtain correct linear equation (allow with no brackets)
M1 **Either** expand bracket and attempt to gather x terms,
or deal correctly with algebraic fraction
 $x = 0.553$ A1 5 Obtain $x = 0.55$, or rounding to this, with no errors seen

5

- 4 (i) $(x^2 - 5)^3 = (x^2)^3 + 3(x^2)^2(-5) + 3(x^2)(-5)^2 + (-5)^3$ M1* Attempt expansion, with product of powers of x^2 and ± 5 ,
at least 3 terms
 $= x^6 - 15x^4 + 75x^2 - 125$ M1* Use at least 3 of binomial coeffs of 1, 3, 3, 1
A1dep* Obtain at least two correct terms, coeffs simplified
A1 4 Obtain fully correct expansion, coeffs simplified
- OR
 $(x^2 - 5)^3 = (x^2 - 5)(x^4 - 10x^2 + 25)$ M2 Attempt full expansion of all 3 brackets
 $= x^6 - 15x^4 + 75x^2 - 125$ A1 Obtain at least two correct terms
A1 Obtain full correct expansion

- (ii) $\int (x^2 - 5)^3 dx = \frac{1}{7} x^7 - 3x^5 + 25x^3 - 125x + c$ M1 Attempt integration of terms of form kx^n
A1√ Obtain at least two correct terms, allow unsimplified coeffs
A1 Obtain $\frac{1}{7} x^7 - 3x^5 + 25x^3 - 125x$
B1 4 $+ c$, and no dx or \int sign

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Mark Scheme

June 2011

<p>5 (i) $2x = 30^\circ, 150^\circ$ $x = 15^\circ, 75^\circ$</p>	<p>M1 Attempt $\sin^{-1} 0.5$, then divide or multiply by 2 A1 Obtain 15° (allow $\pi/12$ or 0.262) A1 3 Obtain 75° (not radians), and no extra solutions in range</p>
<hr/>	
<p>(ii) $2(1 - \cos^2 x) = 2 - \sqrt{3} \cos x$ $2\cos^2 x - \sqrt{3} \cos x = 0$ $\cos x (2\cos x - \sqrt{3}) = 0$ $\cos x = 0, \cos x = \frac{1}{2}\sqrt{3}$ range $x = 90^\circ, x = 30^\circ$</p>	<p>M1 Use $\sin^2 x = 1 - \cos^2 x$ A1 Obtain $2\cos^2 x - \sqrt{3} \cos x = 0$ or equiv (no constant terms) M1 Attempt to solve quadratic in $\cos x$ A1 Obtain 30° (allow $\pi/6$ or 0524), and no extra solns in B1 5 Obtain 90° (allow $\pi/2$ or 1.57), from correct quadratic only SR answer only B1 one correct solution B1 second correct solution, and no others</p>

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<p>6 $\int (3x^2 + a) dx = x^3 + ax + c$ $(-1, 2) \Rightarrow -1 - a + c = 2$ $(2, 17) \Rightarrow 8 + 2a + c = 17$ $a = 2, c = 5$ Hence $y = x^3 + 2x + 5$</p>	<p>M1 Attempt to integrate A1 Obtain at least one correct term, allow unsimplified A1 Obtain $x^3 + ax$ M1 Substitute at least one of $(-1, 2)$ or $(2, 17)$ into integration attempt involving a and c A1 Obtain two correct equations, allow unsimplified M1 Attempt to eliminate one variable from two equations in a and c A1 Obtain $a = 2, c = 5$, from correct equations A1 8 State $y = x^3 + 2x + 5$</p>
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8

<p>7 (i) $f(-2) = -16 + 36 - 22 - 8$ $= -10$</p>	<p>M1 Attempt $f(-2)$, or equiv A1 2 Obtain -10</p>
<hr/>	
<p>(ii) $f(\frac{1}{2}) = \frac{1}{4} + 2\frac{1}{4} + 5\frac{1}{2} - 8 = 0$ AG</p>	<p>M1 Attempt $f(\frac{1}{2})$ (no other method allowed) A1 2 Confirm $f(\frac{1}{2}) = 0$, extra line of working required</p>
<hr/>	
<p>(iii) $f(x) = (2x - 1)(x^2 + 5x + 8)$</p>	<p>M1 Attempt complete division by $(2x - 1)$ or $(x - \frac{1}{2})$ or equiv A1 Obtain $x^2 + 5x + c$ or $2x^2 + 10x + c$ A1 3 State $(2x - 1)(x^2 + 5x + 8)$ or $(x - \frac{1}{2})(2x^2 + 10x + 16)$</p>
<hr/>	
<p>(iv) $f(x)$ has one real root ($x = \frac{1}{2}$) because $b^2 - 4ac = 25 - 32 = -7$ hence quadratic has no real roots as $-7 < 0$,</p>	<p>B1√ State 1 root, following their quotient, ignore reason B1√ 2 Correct calculation, eg discriminant or quadratic formula, following their quotient, or cubic has max at $(-2.15, -9.9)$</p>

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Mark Scheme

June 2011

<p>8 (i) $\frac{1}{2} \times r^2 \times 1.2 = 60$ $r = 10$ $r\theta = 10 \times 1.2 = 12$ perimeter = $10 + 10 + 12 = 32$ cm</p>	<p>M1 Attempt $(\frac{1}{2}) r^2 \theta = 60$ A1 Obtain $r = 10$ B1√ State or imply arc length is $1.2r$, following their r A1 4 Obtain 32</p>
<p>(ii)(a) $u_5 = 60 \times 0.6^4$ $= 7.78$</p>	<p>M1 Attempt u_5 using ar^4, or list terms A1 2 Obtain 7.78, or better</p>
<p>(b) $S_{10} = \frac{60(1-0.6^{10})}{1-0.6}$ $= 149$</p>	<p>M1 Attempt use of correct sum formula for a GP, or sum terms A1 2 Obtain 149, or better (allow 149.0 – 149.2 inclusive)</p>
<p>(c) common ratio is less than 1, so series is convergent and hence sum to infinity exists</p> <p>$S_\infty = \frac{60}{1-0.6}$ $= 150$</p>	<p>B1 series is convergent or $-1 < r < 1$ (allow $r < 1$) or reference to areas getting smaller / adding on less each time</p> <p>M1 Attempt S_∞ using $\frac{a}{1-r}$ A1 3 Obtain $S_\infty = 150$</p> <p>SR B1 only for 150 with no method shown</p>

11

<p>9 (i) </p>	<p>B1 Sketch graph showing exponential growth (both quadrants) B1 2 State or imply (0, 4)</p>
<p>(ii) $4k^x = 20k^2$ $k^x = 5k^2$ $x = \log_k 5k^2$ $x = \log_k 5 + \log_k k^2$ $x = 2\log_k k + \log_k 5$ $x = 2 + \log_k 5$ AG</p> <p>OR $4k^x = 20k^2$ $k^x = 5k^2$ $k^{x-2} = 5$ $x - 2 = \log_k 5$ $x = 2 + \log_k 5$ AG</p>	<p>M1 Equate $4k^x$ to $20k^2$ and take logs (any, or no, base) M1 Use $\log ab = \log a + \log b$ M1 Use $\log a^b = b \log a$ A1 4 Show given answer correctly</p> <p>M1 Attempt to rewrite as single index A1 Obtain $k^{x-2} = 5$ or equiv eg $4k^{x-2} = 20$ M1 Take logs (to any base) A1 Show given answer correctly</p>
<p>(iii) (a) area $\approx \frac{1}{2} \times \frac{1}{2} \times (4k^0 + 8k^{\frac{1}{2}} + 4k^1)$ $\approx 1 + 2k^{\frac{1}{2}} + k$</p>	<p>M1 Attempt y-values at $x = 0, \frac{1}{2}$ and 1, and no others M1 Attempt to use correct trapezium rule, 3 y-values, $h = \frac{1}{2}$ A1 3 Obtain a correct expression, allow unsimplified</p>
<p>(b) $1 + 2k^{\frac{1}{2}} + k = 16$ $(k^{\frac{1}{2}} + 1)^2 = 16$ $k^{\frac{1}{2}} = 3$ $k = 9$</p>	<p>M1 Equate attempt at area to 16 M1 Attempt to solve 'disguised' 3 term quadratic A1 3 Obtain $k = 9$ only</p>

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4723 Core Mathematics 3

- 1 (i) State $y = \sec x$ B1
 (ii) State $y = \cot x$ B1
 (iii) State $y = \sin^{-1} x$ B1 3

3

- 2 Either: State or imply $\int \pi(2x-3)^4 dx$ B1 or unsimplified equiv
 Obtain integral of form $k(2x-3)^5$ M1 any constant k involving π or not
 Obtain $\frac{1}{10}(2x-3)^5$ or $\frac{1}{10}\pi(2x-3)^5$ A1
 Attempt evaluation using 0 and $\frac{3}{2}$ M1 subtraction correct way round
 Obtain $\frac{243}{10}\pi$ A1 5 or exact equiv

- Or: State or imply $\int \pi(2x-3)^4 dx$ B1 or unsimplified equiv
 Expand and obtain integral of order 5 M1 with at least three terms correct
 Ob'n $\frac{16}{5}x^5 - 24x^4 + 72x^3 - 108x^2 + 81x$ A1 with or without π
 Attempt evaluation using (0 and) $\frac{3}{2}$ M1
 Obtain $\frac{243}{10}\pi$ A1 (5) or exact equiv

5

- 3 (i) Attempt use of identity for $\sec^2 \alpha$ M1 using $\pm \tan^2 \alpha \pm 1$
 Obtain $1 + (m+2)^2 - (1+m^2)$ A1 absent brackets implied by subsequent correct working
 Obtain $4m + 4 = 16$ and hence $m = 3$ A1 3

- (ii) Attempt subn in identity for $\tan(\alpha + \beta)$ M1 using $\frac{\pm \tan \alpha \pm \tan \beta}{1 \pm \tan \alpha \tan \beta}$
 Obtain $\frac{5+3}{1-15}$ or $\frac{m+2+m}{1-m(m+2)}$ A1✓ following their m
 Obtain $-\frac{4}{7}$ A1 3 or exact equiv

6

- 4 (i) Obtain $\frac{1}{3}e^{3x} + e^x$ B1
 Substitute to obtain $\frac{1}{3}e^{9a} + e^{3a} - \frac{1}{3}e^{3a} - e^a$ B1 or equiv
 Equate definite integral to 100 and attempt rearrangement M1 as far as $e^{9a} = \dots$
 Introduce natural logarithm M1 using correct process
 Obtain $a = \frac{1}{9}\ln(300 + 3e^a - 2e^{3a})$ A1 5 AG; necessary detail needed

- (ii) Obtain correct first iterate B1 allow for 4 dp rounded or truncated
 Show correct iteration process M1 with at least one more step
 Obtain at least three correct iterates in all A1 allowing recovery after error
 Obtain 0.6309 A1 4 following at least three correct steps; answer required to exactly 4 dp

[0.6 \rightarrow 0.631269 \rightarrow 0.630884 \rightarrow 0.630889]

9

5 (i)	<u>Either:</u> Show correct process for comp'n	M1	correct way round and in terms of x
	Obtain $y = 3(3x + 7) - 2$	A1	or equiv
	Obtain $x = -\frac{19}{9}$	A1	3 or exact equiv; condone absence of $y = 0$
	<u>Or:</u> Use $fg(x) = 0$ to obtain $g(x) = \frac{2}{3}$	B1	
	Attempt solution of $g(x) = \frac{2}{3}$	M1	
	Obtain $x = -\frac{19}{9}$	A1	(3) or exact equiv; condone absence of $y = 0$

(ii)	Attempt formation of one of the equations		
	$3x + 7 = \frac{x - 7}{3}$ or $3x + 7 = x$ or $\frac{x - 7}{3} = x$	M1	or equiv
	Obtain $x = -\frac{7}{2}$	A1	or equiv
	Obtain $y = -\frac{7}{2}$	A1	3 or equiv; following their value of x

(iii)	Attempt solution of modulus equation	M1	squaring both sides to obtain 3-term quadratics or forming linear equation with signs of $3x$ different on each side
	Obtain $-12x + 4 = 42x + 49$ or $3x - 2 = -3x - 7$	A1	or equiv
	Obtain $x = -\frac{5}{6}$	A1	or exact equiv; as final answer
	Obtain $y = \frac{9}{2}$	A1	4 or equiv; and no other pair of answers
10			

6 (i)	Obtain derivative $k(37 + 10y - 2y^2)^{-\frac{1}{2}}f(y)$	M1	any constant k ; any linear function for f
	Obtain $\frac{1}{2}(10 - 4y)(37 + 10y - 2y^2)^{-\frac{1}{2}}$	A1	2 or equiv

(ii)	<u>Either:</u> Sub'te $y = 3$ in expression for $\frac{dx}{dy}$	*M1	
	Take reciprocal of expression/value	*M1	and without change of sign
	Obtain -7 for gradient of tangent	A1	
	Attempt equation of tangent	M1	dep *M *M
	Obtain $y = -7x + 52$	A1	5 and no second equation
	<u>Or:</u> Sub'te $y = 3$ in expression for $\frac{dx}{dy}$	M1	
	Attempt formation of eq'n $x = m'y + c$	M1	where m' is attempt at $\frac{dx}{dy}$
	Obtain $x - 7 = -\frac{1}{7}(y - 3)$	A1	or equiv
	Attempt rearrangement to required form	M1	
	Obtain $y = -7x + 52$	A1	(5) and no second equation
7			

7 (i)	State $R = 10$ Attempt to find value of α Obtain 36.9 or $\tan^{-1} \frac{3}{4}$	B1 M1 A1 3	or equiv implied by correct answer or its complement; allow sin/cos muddles or greater accuracy 36.8699...

(ii)(a)	Show correct process for finding one angle Obtain $(64.16 + 36.87)$ and hence) 101 Show correct process for finding second angle Obtain $(115.84 + 36.87)$ and hence) 153	M1 A1 M1 A1√ 4	 or greater accuracy 101.027... following their value of α ; or greater accuracy 152.711...; and no other between 0 and 360

(b)	Recognise link with part (i) Use fact that maximum and minimum values of sine are 1 and -1 Obtain 60	M1 M1 A1 3	signalled by $40 \dots - 20 \dots$ may be implied; or equiv

8 (i)	Refer to translation and stretch State translation in x direction by 6 State stretch in y direction by 2 [SC: if M0 but one transformation completely correct, give B1]	M1 A1 A1 3	in either order; allow here equiv informal terms such as 'move', ... or equiv; now with correct terminology or equiv; now with correct terminology

(ii)	State $2 \ln(x-6) = \ln x$ Show correct use of logarithm property Attempt solution of 3-term quadratic Obtain 9 only	B1 *M1 M1 A1 4	or $2 \ln(a-6) = \ln a$ or equiv dep *M following correct solution of equation

(iii)	Attempt evaluation of form $k(y_0 + 4y_1 + y_2)$ Obtain $\frac{1}{3} \times (2 \ln 1 + 8 \ln 2 + 2 \ln 3)$ Obtain 2.58	M1 A1 A1 3	any constant k ; maybe with $y_0 = 0$ implied or equiv or greater accuracy 2.5808...

9 (a)	Attempt use of quotient rule Obtain $\frac{(kx^2 + 1)2kx - (kx^2 - 1)2kx}{(kx^2 + 1)^2}$ Obtain correct simplified numerator $4kx$ Equate numerator of first derivative to zero State $x = 0$ <u>or</u> refer to $4kx$ being linear <u>or</u> observe that, with $k \neq 0$, only one sol'n	*M1 A1 A1 M1 A1√ 5	or equiv; allow numerator wrong way round and denominator errors or equiv; with absent brackets implied by subsequent correct working dep *M AG or equiv; following numerator of form $k'kx = 0$, any constant k'

4723

Mark Scheme

(b)	Attempt use of product rule	*M1	
	Obtain $me^{mx}(x^2 + mx) + e^{mx}(2x + m)$	A1	or equiv
	Equate to zero and either factorise with factor e^{mx} or divide through by e^{mx}	M1	dep *M
	Obtain $mx^2 + (m^2 + 2)x + m = 0$ or equiv		
	and observe that e^{mx} cannot be zero	A1	
	Attempt use of discriminant	M1	using correct $b^2 - 4ac$ with their a, b, c
	Simplify to obtain $m^4 + 4$	A1	or equiv
	Observe that this is positive for all m and hence two roots	A1	7 or equiv; AG
			12

4724 Core Mathematics 4

- 1 Long Division For leading term $3x^2$ in quotient B1
 Suff evid of div process (ax^2 , mult back, attempt sub) M1
 (Quotient) = $3x^2 - 4x - 5$ A1
 (Remainder) = $-x + 2$ A1
Identity $3x^4 - x^3 - 3x^2 - 14x - 8 = Q(x^2 + x + 2) + R$ *M1
 $Q = ax^2 + bx + c, R = dx + e$ & attempt ≥ 3 ops. dep*M1 If $a = 3$, this \Rightarrow 1 operation
 $a = 3, b = -4, c = -5$ A1 dep*M1; $Q = ax^2 + bx + c$
 $d = -1, e = 2$ A1
Inspection Use 'Identity' method; if $R = e$, check cf(x) correct before awarding 2nd M1
- 4**
-
- 2 Indefinite Integral Attempt to connect dx & $d\theta$ *M1 Incl $\frac{dx}{d\theta}$ or $\frac{d\theta}{dx}$; not $dx = d\theta$
 Reduce to $\int 1 - \tan^2 \theta (d\theta)$ A1 A0 if $\frac{d\theta}{dx} = \sec^2 \theta$; but allow all following
 Use $\tan^2 \theta = (1, -1) + (\sec^2 \theta, -\sec^2 \theta)$ dep*M1 A marks
 Produce $\int 2 - \sec^2 \theta (d\theta)$ A1
 Correct \int integration of function of type $d + e \sec^2 \theta$ $\sqrt{A1}$ including $d = 0$
 EITHER Attempt limits change (allow degrees here) M1 (This is 'limits' aspect; the
 OR Attempt integ, re-subst & use original ($\sqrt{3}, 1$) integ need not be accurate)
 $\frac{1}{6}\pi - \sqrt{3} + 1$ isw Exact answer required A1
- 7**

3 (i)	$(1 + \frac{x}{a})^{-2} = 1 + (-2)\frac{x}{a} + \frac{-2 \cdot -3}{2}(\frac{x}{a})^2 + \dots$	M1	Check 3 rd term; accept $\frac{x^2}{a}$
	$= 1 - \frac{2x}{a} + \dots$ or $1 + (-\frac{2x}{a})$	B1	or $1 - 2xa^{-1}$ (Ind of M1)
	$\dots + \frac{3x^2}{a^2} + \dots$ (or $3(\frac{x}{a})^2$ or $3x^2a^{-2}$)	A1	Accept $\frac{6}{2}$ for 3
	$(a+x)^{-2} = \frac{1}{a^2} \{ \text{their expansion of } (1 + \frac{x}{a})^{-2} \}$ mult out	√A1 4	$\frac{1}{a^2} - \frac{2x}{a^3} + \frac{3x^2}{a^4}$; accept ega^{-2}

(ii)	Mult out $(1-x)$ (their exp) to produce all terms/cfs (x^2)	M1	Ignore other terms
	Produce $\frac{3}{a^2} + \frac{2}{a} (=0)$ or $\frac{3}{a^4} + \frac{2}{a^3} (=0)$ or AEF	A1	Accept x^2 if in both terms
	$a = -\frac{3}{2}$ www seen anywhere in (i) or (ii)	A1 3	Disregard any ref to $a = 0$

7

4 (i)	Differentiate as a product, $u dv + v du$	M1	or as 2 separate products
	$\frac{d}{dx}(\sin 2x) = 2 \cos 2x$ or $\frac{d}{dx}(\cos 2x) = -2 \sin 2x$	B1	
	$e^x(2 \cos 2x + 4 \sin 2x) + e^x(\sin 2x - 2 \cos 2x)$	A1	terms may be in diff order
	Simplify to $5 e^x \sin 2x$ www	A1 4	Accept $10e^x \sin x \cos x$

(ii)	Provided result (i) is of form $k e^x \sin 2x$, k const		
	$\int e^x \sin 2x dx = \frac{1}{k} e^x (\sin 2x - 2 \cos 2x)$	B1	
	$[e^x (\sin 2x - 2 \cos 2x)]_0^{\frac{1}{4}\pi} = e^{\frac{1}{4}\pi} + 2$	B1	
	$\frac{1}{5} (e^{\frac{1}{4}\pi} + 2)$	B1 3	Exact form to be seen

SR Although 'Hence', award M2 for double integration by parts and solving + A1 for correct answer.

7

- 5 (i) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ aef used M1
- $= \frac{4t + 3t^2}{2 + 2t}$ A1
- Attempt to find t from one/both equations M1 or diff (ii) cartesian eqn \rightarrow M1
- State/imply $t = -3$ is only solution of both equations A1 subst (3,-9), solve for $\frac{dy}{dx} \rightarrow$ M1
- Gradient of curve = $-\frac{15}{4}$ or $\frac{-15}{4}$ or $\frac{15}{-4}$ A1 **5** grad of curve = $-\frac{15}{4} \rightarrow$ A1
- [SR If $t = 1$ is given as solution & not disqualified, award A0 + $\sqrt{A1}$ for grad = $-\frac{15}{4}$ & $\frac{7}{4}$;
- If $t = 1$ is given/used as only solution, award A0 + $\sqrt{A1}$ for grad = $\frac{7}{4}$]

- (ii) $\frac{y}{x} = t$ B1
- Substitute into either parametric eqn M1
- Final answer $x^3 = 2xy + y^2$ A2 **4**
- [SR Any correct unsimplified form (involving fractions or common factors) \rightarrow A1]

9

- 6 (i) $4x \equiv A(x-3)^2 + B(x-3)(x-5) + C(x-5)$ M1
- $A = 5$ A1 'cover-up' rule, award B1
- $B = -5$ A1
- $C = -6$ A1 **4** 'cover-up' rule, award B1
- Cands adopting other alg. manip. may be awarded M1 for a full satis method + 3 @ A1

- (ii) $\int \frac{A}{x-5} dx = A \ln(5-x)$ or $A \ln|5-x|$ or $A \ln|x-5|$ $\sqrt{B1}$ but not $A \ln(x-5)$
- $\int \frac{B}{x-3} dx = B \ln(3-x)$ or $B \ln|3-x|$ or $B \ln|x-3|$ $\sqrt{B1}$ but not $B \ln(x-3)$
- If candidate is awarded B0,B0, then award SR $\sqrt{B1}$ for $A \ln(x-5)$ **and** $B \ln(x-3)$
- $\int \frac{C}{(x-3)^2} dx = -\frac{C}{x-3}$ $\sqrt{B1}$
- $5 \ln \frac{3}{4} + 5 \ln 2$ aef, isw $\sqrt{A \ln \frac{3}{4} - B \ln 2}$ $\sqrt{B1}$ Allow if SR B1 awarded
- -3 $\sqrt{\frac{1}{2}C}$ $\sqrt{B1}$ **5**
- [Mark at earliest correct stage & isw; no ln 1] 9

4724

Mark Scheme

June 2015

- 7 (i) Attempt scalar prod $\{\mathbf{u} \cdot (4\mathbf{i} + \mathbf{k})$ or $\mathbf{u} \cdot (4\mathbf{i} + 3\mathbf{j} + 2\mathbf{k})\} = 0$ M1 where \mathbf{u} is the given vector
- Obtain $\frac{12}{13} + c = 0$ or $\frac{12}{13} + 3b + 2c = 0$ A1
- $c = -\frac{12}{13}$ A1
- $b = \frac{4}{13}$ A1 cao No ft
- Evaluate $\left(\frac{3}{13}\right)^2 + (\text{their } b)^2 + (\text{their } c)^2$ M1 Ignore non-mention of $\sqrt{\quad}$
- Obtain $\frac{9}{169} + \frac{144}{169} + \frac{16}{169} = 1$ AG A1 6 Ignore non-mention of $\sqrt{\quad}$

- (ii) Use $\cos \theta = \frac{\mathbf{x} \cdot \mathbf{y}}{\|\mathbf{x}\| \|\mathbf{y}\|}$ M1
- Correct method for finding scalar product M1
- 36° (35.837653...) Accept 0.625 (rad) A1 3 From $\frac{18}{\sqrt{17}\sqrt{29}}$
- SR If $4\mathbf{i} + \mathbf{k} = (4, 1, 0)$ in (i) & (ii), mark as scheme but allow final A1 for 31° (31.160968) or 0.544

9

- 8 (i) $\frac{d}{dx}(y^2) = 2y \frac{dy}{dx}$ B1
- $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$ used on $(-7)xy$ M1
- $\frac{d}{dx}(14x^2 - 7xy + y^2) = 28x - 7x \frac{dy}{dx} - 7y + 2y \frac{dy}{dx}$ A1 (= 0)
- $2y \frac{dy}{dx} - 7x \frac{dy}{dx} = 7y - 28x \rightarrow \frac{dy}{dx} = \frac{28x - 7y}{7x - 2y}$ www AG A1 4 As AG, intermed step nec

- (ii) Subst $x = 1$ into eqn curve & solve quadratic eqn in y M1 ($y = 3$ or 4)
- Subst $x = 1$ and (one of) their y -value(s) into given $\frac{dy}{dx}$ M1 ($\frac{dy}{dx} = 7$ or 0)
- Find eqn of tgt, with their $\frac{dy}{dx}$, going through (1, their y) *M1 using (one of) y value(s)
- Produce either $y = 7x - 4$ or $y = 4$ A1
- Solve simultaneously their two equations dep*M1 provided they have two
- Produce $x = \frac{8}{7}$ A1 6

10

9 (i)	$\frac{20}{k_1}$ (seconds)	B1 1
<hr/>		
(ii)	$\frac{d\theta}{dt} = -k_2(\theta - 20)$	B1 1
<hr/>		
(iii)	Separate variables or invert each side Correct int of each side (+ c) Subst $\theta = 60$ when $t = 0$ into eqn containing 'c'	M1 Correct eqn or very similar A1,A1 for each integration M1 or $\theta = 60$ when $t =$ their (i)
	c (or $-c$) = $\ln 40$ or $\frac{1}{k_2} \ln 40$ or $\frac{1}{k_2} \ln 40k_2$	A1 Check carefully their 'c'
	Subst their value of c and $\theta = 40$ back into equation	M1 Use scheme on LHS
	$t = \frac{1}{k_2} \ln 2$	A1 Ignore scheme on LHS
	Total time = $\frac{1}{k_2} \ln 2$ + their (i) (seconds)	$\sqrt{A1}$ 8

SR If the negative sign is omitted in part (ii), allow all marks in (iii) with $\ln 2$ replaced by $\ln \frac{1}{2}$.

SR If definite integrals used, allow M1 for eqn where $t = 0$ and $\theta = 60$ correspond; a second M1 for eqn where $t = t$ and $\theta = 40$ correspond & M1 for correct use of limits. Final answer scores 2.

4725 Further Pure Mathematics 1

1.	$984390625 - 25502500 = 958888125$	B1 M1 A1	3 3	State correct value of S_{250} or S_{100} Subtract $S_{250} - S_{100}$ (or S_{101} or S_{99}) Obtain correct exact answer
2.	$3a + 5b = 1, a + 2b = 1$ $a = -3, b = 2$	M1 M1 A1 A1	4 4	Obtain a pair of simultaneous equations Attempt to solve Obtain correct answers.
3.	(i) $11 - 29i$ (ii) $1 + 41i$	B1 B1 B1 B1	2 2 4	Correct real and imaginary parts Correct real and imaginary parts
4.	Either $p + q = -1, pq = -8$ $\frac{p+q}{pq}$ $-\frac{7}{8}$ Or $\frac{1}{p} + \frac{1}{q} = 8$ $p + q = 1$ $-\frac{7}{8}$ Or $\frac{-1 \pm \sqrt{33}}{2}$ $-\frac{7}{8}$	B1 B1 M1 A1 B1 B1 M1 A1 M1 A1 M1 A1 M1 A1	4 4	Both values stated or used Correct expression seen Use their values in their expression Obtain correct answer Substitute $x = \frac{1}{u}$ and use new quadratic Correct value stated Use their values in given expression Obtain correct answer Find roots of given quadratic equation Correct values seen Use their values in given expression Obtain correct answer
5.	(i) $u^3 = \{(-)(5u + 7)\}^2$ $u^3 - 25u^2 - 70u - 49 = 0$ (ii) -70	M1 A1 A1 M1 A1 ft	3 3 2 5	Use given substitution and rearrange Obtain correct expression, or equivalent Obtain correct final answer Use coefficient of u of their cubic or identity connecting the symmetric functions and substitute values from given equation Obtain correct answer

6.	(i) $3\sqrt{2}, -\frac{\pi}{4}$ or -45° AEF	B1 B1	2	State correct answers
	(ii)(a)	B1B1	3	Circle, centre (3, -3), through O ft for $(\pm 3, \pm 3)$ only
	(ii)(b)	B1 B1 B1	3	Straight line with +ve slope, through (3, -3) or their centre Half line only starting at centre
	(iii)	B1ft B1ft B1ft	3	Area above horizontal through a , below (ii) (b) Outside circle
			11	
7.	(i)	M1 A1	2	Show that terms cancel in pairs Obtain given answer correctly
	(ii)	M1 A1	2	Attempt to expand and simplify Obtain given answer correctly
	(iii)	B1 B1		Correct $\sum r$ stated $\sum 1 = n$
	$(n+1)^4 - 1 - n(n+1)(2n+1) - 2n(n+1) - n$	M1* *DM1 A1		Consider sum of 4 separate terms on RHS Required sum is LHS – 3 terms Correct unsimplified expression
	$4\sum_{r=1}^n r^3 = n^2(n+1)^2$	A1	6	Obtain given answer correctly
			10	
8.	(i)	B1 B1 B1	3	Find coordinates (0, 0) (3, 1) (2, 1) (5, 2) found Accurate diagram sketched
	(ii) $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$	B1 B1	2	Each column correct
	(iii) <i>Either</i> $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$	B1 M1 A1ft		Correct inverse for their (ii) stated Post multiply C by inverse of (ii) Correct answer found
	Or	M1 A2ft		Set up 4 equations for elements from correct matrix multiplication All elements correct, -1 each error
		B1 B1 B1	6	Shear, x axis invariant or parallel to x -axis eg image of (1, 1) is (3, 1)
			11	SR allow s.f. 2 or shearing angle of correct angle to appropriate axis

9.	(i) $a \begin{vmatrix} a & 1 \\ 1 & 2 \end{vmatrix} - \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 1 & a \\ 1 & 1 \end{vmatrix}$ $2a^2 - 2a$ (ii) $a = 0$ or 1 (iii) (a) (b)	M1 A1 A1 M1 A1ft A1ft B1 B1 B1 B1	3 3 4 10	Correct expansion process shown Obtain correct unsimplified expression Obtain correct answer Equate their det to 0 Obtain correct answers, ft solving a quadratic Equations consistent, but non unique solutions Correct equations seen & inconsistent, no solutions
10.	i) $u_2 = 7$ $u_3 = 19$ (ii) $u_n = 2(3^{n-1}) + 1$ (iii) $u_{n+1} = 3(2(3^{n-1}) + 1) - 2$ $u_{n+1} = 2(3^n) + 1$	M1 A1 A1 M1 A1 B1ft M1 A1 A1 B1	3 2 5 10	Attempt to find next 2 terms Obtain correct answers Show given result correctly Expression involving a power of 3 Obtain correct answer Verify result true when $n = 1$ or $n = 2$ Expression for u_{n+1} using recurrence relation Correct unsimplified answer Correct answer in correct form Statement of induction conclusion

4726 Further Pure Mathematics 2

1(i)	Attempt area = $\pm \Sigma(0.3y)$ for at least three y values	M1	May be implied
	Get 1.313(1..) or 1.314	A1	Or greater accuracy
(ii)	Attempt \pm sum of areas (4 or 5 values)	M1	May be implied
	Get 0.518(4..)	A1	Or greater accuracy
			SC
			If answers only seen, 1.313(1..) or 1.314 B2
			0.518(4..) B2
		-1.313(1..) or -1.314 B1	
		-0.518(4..) B1	
	Or		
	Attempt answer to part (i) – final rectangle	M1	
	Get 0.518(4..)	A1	
(iii)	Decrease width of strips	B1	Use more strips or equivalent
2	Attempt to set up quadratic in x	M1	Must be quadratic; = 0 may be implied
	Get $x^2(y-1) - x(2y+1) + (y-1) = 0$	A1	
	Use $b^2 \geq 4ac$ for real x on their quadratic	M1	Allow =, >, <, \leq here; may be implied
	Clearly solve to AG	A1	If other (in)equalities used, the step to AG must be clear
			SC
			Reasonable attempt to diff. using prod/quot rule M1
			Solve correct $dy/dx=0$ to get $x=-1, y=1/4$ A1
		Attempt to justify inequality e.g. graph or to show $d^2y/dx^2 > 0$ M1	
		Clearly solve to AG A1	
3(i)	Reasonable attempt at chain rule	M1	Product in answer
	Reasonable attempt at product/quotient rule	M1	Sum of two parts
	Correctly get $f'(0) = 1$	A1	
	Correctly get $f''(0) = 1$	A1	SC
			Use of $\ln y = \sin x$ follows same scheme
(ii)	Reasonable attempt at Maclaurin with their values	M1	In $af(0) + bf'(0)x + cf''(0)x^2$
	Get $1 + x + \frac{1}{2}x^2$	A1✓	From their $f(0), f'(0), f''(0)$ in a correct Maclaurin; all non-zero terms
4	Attempt to divide out.	M1	Or $A+B/(x-2)+(Cx(+D))/(x^2+4)$; allow $A=1$ and/or $B=1$ quoted
	Get $x^3 =$	M1	Allow ✓ mark from their Part Fract; allow $D=0$ but not $C=0$
	$A(x-2)(x^2+4)+B(x^2+4)+(Cx+D)(x-2)$		
	State/derive/quote $A=1$	A1	
	Use x values and/or equate coeff	M1	To potentially get all their constants

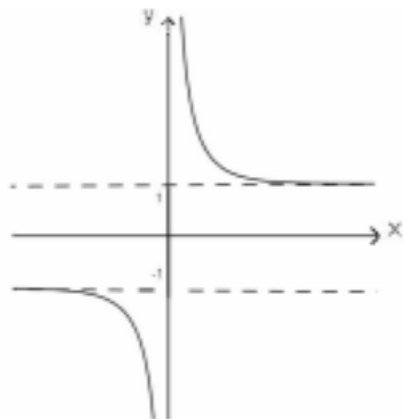
4726

Mark Scheme

June 200

	Get $B=1, C=1, D=-2$	A1	For one other correct from cwo
		A1	For all correct from cwo
5(i)	Derive/quote $d\theta=2dt/(1+t^2)$	B1	May be implied
	Replace their $\cos \theta$ and their $d\theta$, both in terms of t	M1	Not $d\theta = dt$
	Clearly get $\int(1-t^2)/(1+t^2) dt$ or equiv	A1	Accept limits of t quoted here
	Attempt to divide out	M1	Or use AG to get answer above
	Clearly get/derive AG	A1	
		SC	
		Derive $d\theta = 2\cos^2 \frac{1}{2}\theta dt$	B1
		Replace $\cos \theta$ in terms of half-angles and their $d\theta (\neq dt)$	M1
		Get $\int 2\cos^2 \frac{1}{2}\theta - 1 dt$ or	
		$\int 1 - 1/2\cos^2 \frac{1}{2}\theta .2/(1+t^2) dt$	A1
		Use $\sec^2 \frac{1}{2}\theta = 1+t^2$	M1
		Clearly get/derive AG	A1
(ii)	Integrate to $a \tan^{-1} bt - t$	M1	
	Get $\frac{1}{2}\pi - 1$	A1	
6	Get $k \sinh^{-1} k_1 x$	M1	For either integral; allow attempt at ln version here
	Get $\frac{1}{3} \sinh^{-1} \frac{3}{4}x$	A1	Or ln version
	Get $\frac{1}{2} \sinh^{-1} \frac{2}{3}x$	A1	Or ln version
	Use limits in their answers	M1	
	Attempt to use correct ln laws to set up a solvable equation in a	M1	
	Get $a = 2^{\frac{1}{3}} \cdot 3^{\frac{1}{2}}$	A1	Or equivalent

7(i)



- B1 y -axis asymptote; equation may be implied if clear
- B1 Shape
- B1 $y = \pm 1$ asymptotes; may be implied if seen as on graph

(ii) Reasonable attempt at product rule, giving two terms

M1

Use correct Newton-Raphson at least once with their $f'(x)$ to produce an x_2

M1 May be implied

Get $x_2 = 2.0651$

A1√

Get $x_3 = 2.0653, x_4 = 2.0653$

A1

One correct at any stage if reasonable cao; or greater accuracy which rounds

(iii) Clearly derive $\coth x = \frac{1}{2}x$

B1

AG; allow derivation from AG
Two roots onlyAttempt to find second root e.g. symmetry
Get ± 2.0653

M1

 \pm their iteration in part (ii)

A1√

8(i) (a) Get $\frac{1}{2}(e^{\ln a} + e^{-\ln a})$
Use $e^{\ln a} = a$ and $e^{-\ln a} = 1/a$
Clearly derive AG

M1

M1

A1

(b) Reasonable attempt to multiply out their attempts at exponential definitions of cosh and sinh

M1

4 terms in each

Correct expansion seen as $e^{(x+y)}$ etc.

A1

Clearly tidy to AG

A1

With $e^{(x-y)}$ seen or implied(ii) Use $x = y$ and $\cosh 0 = 1$ to get AG

B1

(iii) Attempt to expand and equate coefficients

M1

 $13 = R \cosh \ln a = R(a^2+1)/2a$
 $5 = R \sinh \ln a = R(a^2-1)/2a$ Attempt to eliminate R (or a) to set up a solvable equation in a (or R)

M1

SC

If exponential definitions used,
 $8e^x + 18e^{-x} = Re^x/a + Rae^{-x}$ and same scheme followsGet $a = \sqrt[3]{2}$ (or $R = 12$)

A1

Replace for a (or R) in relevant equation to set up solvable equation in R (or a)

M1

Get $R=12$ (or $a = \sqrt[3]{2}$)

A1

Ignore if $a = \sqrt[3]{2}$ also given(iv) Quote/derive $(\ln \sqrt[3]{2}, 12)$

B1√

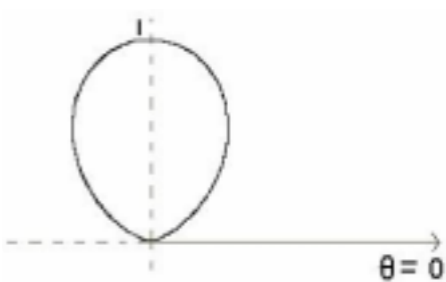
On their R and a

B1√

9(i) Use $\sin \theta \cdot \sin^{n-1} \theta$ and parts

M1

Reasonable attempt with 2 parts, one yet to be integrated

Get	A1	Signs need to be carefully considered
$-\cos\theta.\sin^{n-1}\theta+(n-1)\sin^{n-2}\theta.\cos^2\theta d\theta$		
Replace $\cos^2 = 1 - \sin^2$	M1	
Clearly use limits and get AG	A1	
(ii) (a) Solve for $r=0$ for at least one θ	M1	θ need not be correct
Get $(\theta) = 0$ and π	A1	Ignore extra answers out of range
	B1	General shape (symmetry stated or approximately seen)
		
	B1	Tangents at $\theta=0, \pi$ and max r seen
(b) Correct formula used; correct r	M1	May be $\int r^2 d\theta$ with correct limits
Use $6I_6 = 5I_4, 4I_4 = 3I_2$	M1	At least one
Attempt I_0 (or I_2)	M1	($I_0 = \frac{1}{2}\pi$)
Replace their values to get I_6	M1	
Get $5\pi/32$	A1	
Use symmetry to get $5\pi/32$	A1	May be implied but correct use of limits must be given somewhere in answer
Or		
Correct formula used; correct r	M1	
Reasonable attempt at formula		
$(2i\sin\theta)^6 = (z - 1/z)^6$	M1	
Attempt to multiply out both sides		
(7 terms)	M1	
Get correct expansion	A1	
Convert to trig. equivalent and integrate their expression	M1	cwo
Get $5\pi/32$	A1	
Or		
Correct formula used; correct r	M1	
Use double-angle formula and attempt to cube (4 terms)	M1	
Get correct expression	A1	
Reasonable attempt to put $\cos^2 2\theta$ into integrable form and integrate	M1	
Reasonable attempt to integrate $\cos^3 2\theta$ as e.g. $\cos^2 2\theta.\cos 2\theta$	M1	cwo
Get $5\pi/32$	A1	

4727 Further Pure Mathematics 3

1	$\left(\frac{1}{2}\sqrt{3} + \frac{1}{2}i\right)^{\frac{1}{3}} = \left(\cos\frac{1}{6}\pi + i\sin\frac{1}{6}\pi\right)^{\frac{1}{3}}$	B1	For $\arg z = \frac{1}{6}\pi$ seen or implied
	$= \cos\frac{1}{18}\pi + i\sin\frac{1}{18}\pi,$	M1	For dividing $\arg z$ by 3
	$\cos\frac{13}{18}\pi + i\sin\frac{13}{18}\pi,$	A1	For any one correct root
	$\cos\frac{25}{18}\pi + i\sin\frac{25}{18}\pi$	A1 4	For 2 other roots and no more in range $0 \leq \theta < 2\pi$
4			
2 (i)	$\frac{1}{5}e^{-\frac{1}{3}\pi i}$	B1 1	For stating correct inverse in the form $re^{i\theta}$
	(ii) $r_1e^{i\theta} \times r_2e^{i\phi} = r_1r_2e^{i(\theta+\phi)}$	M1	For stating 2 distinct elements multiplied
		A1 2	For showing product of correct form
(iii)	$Z^2 = e^{2i\gamma}$	B1	For $e^{2i\gamma}$ seen or implied
	$\Rightarrow e^{2i\gamma-2\pi i}$	B1 2	For correct answer. aef
5			
3 (i)	$[6-4\lambda, -7+8\lambda, -10+7\lambda]$ on l	B1	For point on l seen or implied
	$\Rightarrow 3(6-4\lambda) - 4(-7+8\lambda) - 2(-10+7\lambda) = 8$	M1	For substituting into equation of p
	$\Rightarrow \lambda = 1 \Rightarrow (2, 1, -3)$	A1 3	For correct point. Allow position vector
(ii)	METHOD 1		
	$\mathbf{n} = [-4, 8, 7] \times [3, -4, -2]$	M1*	For direction of l and normal of p seen
		M1	For attempting to find $\mathbf{n}_1 \times \mathbf{n}_2$
		(*dep)	
	$\mathbf{n} = k[12, 13, -8]$	A1	For correct vector
	$(2, 1, -3)$ OR $(6, -7, -10)$	M1	For finding scalar product of their point on l with their attempt at \mathbf{n} , or equivalent
	$\Rightarrow 12x + 13y - 8z = 61$	A1 5	For correct equation, aef cartesian
	METHOD 2		
	$\mathbf{r} = [2, 1, -3]$ OR $[6, -7, -10]$ $+ \lambda[-4, 8, 7] + \mu[3, -4, -2]$	M1 A1√	For stating eqn of plane in parametric form (may be implied by next stage), using $[2, 1, -3]$ (ft from (i)) Or $[6, -7, -10]$, \mathbf{n}_1 and \mathbf{n}_2 (as above)
	$x = 2 - 4\lambda + 3\mu$	M1	For writing as 3 linear equations
$y = 1 + 8\lambda - 4\mu$	M1	For attempting to eliminate λ and μ	
$z = -3 + 7\lambda - 2\mu$			
$\Rightarrow 12x + 13y - 8z = 61$	A1	For correct equation aef cartesian	
METHOD 3			
$3(6+3\mu) - 4(-7-4\mu) - 2(-10-2\mu) = 8$	M1	For finding foot of perpendicular from point on l to p	
$\Rightarrow \mu = -2 \Rightarrow (0, 1, -6)$	A1	For correct point or position vector	
From 3 points $(2, 1, -3)$, $(6, -7, -10)$, $(0, 1, -6)$,			
$\mathbf{n} =$ vector product of 2 of $[2, 0, 3]$, $[6, -8, -4]$, $[-4, 8, 7]$	M1	Use vector product of 2 vectors in plane	
$\Rightarrow \mathbf{n} = k[12, 13, -8]$			
$(2, 1, -3)$ OR $(6, -7, -10)$	M1	For finding scalar product of their point on l with their attempt at \mathbf{n} , or equivalent	
$\Rightarrow 12x + 13y - 8z = 61$	A1	For correct equation aef cartesian	
8			

4 (i)	IF $\int \frac{1}{1-x^2} dx = e^{\frac{1}{2} \ln \frac{1+x}{1-x}} = \left(\frac{1+x}{1-x}\right)^{\frac{1}{2}}$	M1	For IF stated or implied. Allow \pm and omission of
		A1 2	dx For integration and simplification to AG (intermediate step must be seen)
(ii)	$\frac{d}{dx} \left(y \left(\frac{1+x}{1-x} \right)^{\frac{1}{2}} \right) = (1+x)^{\frac{1}{2}}$ $y \left(\frac{1+x}{1-x} \right)^{\frac{1}{2}} = \frac{2}{3} (1+x)^{\frac{3}{2}} + c$ $(0, 2) \Rightarrow 2 = \frac{2}{3} + c \Rightarrow c = \frac{4}{3}$ $y = \frac{2}{3} (1+x) (1-x)^{\frac{1}{2}} + \frac{4}{3} \left(\frac{1-x}{1+x} \right)^{\frac{1}{2}}$	M1*	For multiplying both sides by IF
		M1	For integrating RHS to $k(1+x)^n$
		A1	For correct equation (including $+c$) In either order:
		M1 (*dep)	For substituting $(0, 2)$ into their GS (including $+c$)
		M1 (*dep)	For dividing solution through by IF, including dividing c or their numerical value for c
A1 6	For correct solution aef (even unsimplified) in form $y = f(x)$		

8

5 (i)	$m^2 - 6m + 9 (= 0) \Rightarrow m = 3$ CF = $(A + Bx)e^{3x}$	M1	For attempting to solve correct auxiliary equation
		A1	For correct m
		A1 3	For correct CF
(ii)	ke^{3x} and kxe^{3x} both appear in CF	B1 1	For correct statement
(iii)	$y = kx^2e^{3x} \Rightarrow y' = 2kxe^{3x} + 3kx^2e^{3x}$ $\Rightarrow y'' = 2ke^{3x} + 12kxe^{3x} + 9kx^2e^{3x}$ \Rightarrow $ke^{3x}(2 + 12x + 9x^2 - 12x - 18x^2 + 9x^2) = e^{3x}$ $\Rightarrow k = \frac{1}{2}$	M1	For differentiating kx^2e^{3x} twice
		A1	For correct y' aef
		A1	For correct y'' aef
		M1	For substituting y'', y', y into DE
		A1 5	For correct k

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6 (i)	METHOD 1			
	$\mathbf{n}_1 = [1, 1, 0] \times [1, -5, -2]$	M1	For attempting to find vector product of the pair of direction vectors	
	$= [-2, 2, -6] = k[1, -1, 3]$	A1	For correct \mathbf{n}_1	
	Use (2, 2, 1)	M1	For substituting a point into equation	
	$\Rightarrow \mathbf{r} \cdot [-2, 2, -6] = -6 \Rightarrow \mathbf{r} \cdot [1, -1, 3] = 3$	A1	4 For correct equation. aef in this form	
METHOD 2				
	$x = 2 + \lambda + \mu$	M1	For writing as 3 linear equations	
	$y = 2 + \lambda - 5\mu$	M1	For attempting to eliminate λ and μ	
	$z = 1 - 2\mu$			
	$\Rightarrow x - y + 3z = 3$	A1	For correct cartesian equation	
	$\Rightarrow \mathbf{r} \cdot [1, -1, 3] = 3$	A1	For correct equation. aef in this form	
(ii)	For $\mathbf{r} = \mathbf{a} + t\mathbf{b}$			
	METHOD 1			
	$\mathbf{b} = [1, -1, 3] \times [7, 17, -3]$	M1	For attempting to find $\mathbf{n}_1 \times \mathbf{n}_2$	
	$= k[2, -1, -1]$	A1√	For a correct vector. ft from \mathbf{n}_1 in (i)	
	e.g. x, y or $z = 0$ in $\begin{cases} x - y + 3z = 3 \\ 7x + 17y - 3z = 21 \end{cases}$	M1	For attempting to find a point on the line	
	$\Rightarrow \mathbf{a} = \left[0, \frac{3}{2}, \frac{3}{2}\right]$ OR $[3, 0, 0]$ OR $[1, 1, 1]$	A1√	For a correct vector. ft from equation in (i) SR a correct vector may be stated without working	
	Line is (e.g.) $\mathbf{r} = [1, 1, 1] + t[2, -1, -1]$	A1√	5 For stating equation of line ft from \mathbf{a} and \mathbf{b} SR for $\mathbf{a} = [2, 2, 1]$ stated award M0	
	METHOD 2			
	Solve $\begin{cases} x - y + 3z = 3 \\ 7x + 17y - 3z = 21 \end{cases}$	M1	In either order: For attempting to solve equations	
	by eliminating one variable (e.g. z)			
	Use parameter for another variable (e.g. x) to find other variables in terms of t	M1	For attempting to find parametric solution	
	(eg) $y = \frac{3}{2} - \frac{1}{2}t, z = \frac{3}{2} - \frac{1}{2}t$	A1√	For correct expression for one variable	
	A1√	For correct expression for the other variable ft from equation in (i) for both		
Line is (eg) $\mathbf{r} = \left[0, \frac{3}{2}, \frac{3}{2}\right] + t[2, -1, -1]$	A1√	For stating equation of line. ft from parametric solutions		
METHOD 3				
eg x, y or $z = 0$ in $\begin{cases} x - y + 3z = 3 \\ 7x + 17y - 3z = 21 \end{cases}$	M1	For attempting to find a point on the line		
$\Rightarrow \mathbf{a} = \left[0, \frac{3}{2}, \frac{3}{2}\right]$ OR $[3, 0, 0]$ OR $[1, 1, 1]$	A1√	For a correct vector. ft from equation in (i) SR a correct vector may be stated without working SR for $\mathbf{a} = [2, 2, 1]$ stated award M0		
eg $[3, 0, 0] - [1, 1, 1]$	M1	For finding another point on the line and using it with the one already found to find \mathbf{b}		
$\mathbf{b} = k[2, -1, -1]$	A1√	For a correct vector. ft from equation in (i)		
Line is (eg) $\mathbf{r} = [1, 1, 1] + t[2, -1, -1]$	A1√	For stating equation of line. ft from \mathbf{a} and \mathbf{b}		

6 (ii) contd	METHOD 4		
A point on Π_1 is [2 + λ + μ , 2 + λ - 5 μ , 1 - 2 μ]	M1	For using parametric form for Π_1 and substituting into Π_2	
On $\Pi_2 \Rightarrow$ [2 + λ + μ , 2 + λ - 5 μ , 1 - 2 μ] \cdot [7, 17, -3] = 21	A1	For correct unsimplified equation	
$\Rightarrow \lambda - 3\mu = -1$	A1	For correct equation	
Line is (e.g.) $\mathbf{r} = [2, 2, 1] + (3\mu - 1)[1, 1, 0] + \mu[1, -5, -2]$	M1	For substituting into Π_1 for λ or μ	
$\Rightarrow \mathbf{r} = [1, 1, 1]$ or $\left[\frac{7}{3}, \frac{1}{3}, \frac{1}{3}\right] + t[2, -1, -1]$	A1	For stating equation of line	
9			
7 (i)	$\cos 3\theta + i \sin 3\theta = c^3 + 3ic^2s - 3cs^2 - is^3$	M1	For using de Moivre with $n = 3$
$\Rightarrow \cos 3\theta = c^3 - 3cs^2$ and $\sin 3\theta = 3c^2s - s^3$	A1	For both expressions in this form (seen or implied) SR For expressions found without de Moivre M0 A0	
$\Rightarrow \tan 3\theta = \frac{3c^2s - s^3}{c^3 - 3cs^2}$	M1	For expressing $\frac{\sin 3\theta}{\cos 3\theta}$ in terms of c and s	
$\tan 3\theta = \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta} = \frac{\tan \theta (3 - \tan^2 \theta)}{1 - 3 \tan^2 \theta}$	A1 4	For simplifying to AG	
(ii) (a)	$\theta = \frac{1}{12}\pi \Rightarrow \tan 3\theta = 1$ $\Rightarrow 1 - 3t^2 = t(3 - t^2) \Rightarrow$ $t^3 - 3t^2 - 3t + 1 = 0$	B1 1	For both stages correct AG
(b)	$(t+1)(t^2 - 4t + 1) = 0$ $\Rightarrow (t = -1), t = 2 \pm \sqrt{3}$ - sign for smaller root \Rightarrow $\tan \frac{1}{12}\pi = 2 - \sqrt{3}$	M1 A1 A1 A1 4	For attempt to factorise cubic For correct factors For correct roots of quadratic For choice of - sign and correct root AG
(iii)	$dt = (1 + t^2) d\theta$ $\Rightarrow \int_0^{\frac{1}{12}\pi} \tan 3\theta d\theta$ $= \left[\frac{1}{3} \ln(\sec 3\theta) \right]_0^{\frac{1}{12}\pi} = \frac{1}{3} \ln(\sec \frac{1}{4}\pi)$ $= \frac{1}{3} \ln \sqrt{2} = \frac{1}{6} \ln 2$	B1 B1 M1 M1 A1 5	For differentiation of substitution and use of $\sec^2 \theta = 1 + \tan^2 \theta$ For integral with correct θ limits seen For integrating to $k \ln(\sec 3\theta)$ OR $k \ln(\cos 3\theta)$ For substituting limits and $\sec \frac{1}{4}\pi = \sqrt{2}$ OR $\cos \frac{1}{4}\pi = \frac{1}{\sqrt{2}}$ seen For correct answer aef
14			

8 (i)	$a^2 = (ap)^2 = apap \Rightarrow a = pap$	B1	For use of given properties to obtain AG																									
	$p^2 = (ap)^2 = apap \Rightarrow p = apa$	B1 2	For use of given properties to obtain AG SR allow working from AG to obtain relevant properties																									
(ii)	$(p^2)^2 = p^4 = e \Rightarrow \text{order } p^2 = 2$	B1	For correct order with no incorrect working seen																									
	$(a^2)^2 = (p^2)^2 = e \Rightarrow \text{order } a = 4$	B1	For correct order with no incorrect working seen																									
	$(ap)^4 = a^4 = e \Rightarrow \text{order } ap = 4$	B1	For correct order with no incorrect working seen																									
	$(ap^2)^2 = ap^2ap^2 = ap \cdot a \cdot p = a^2$	M1	For relevant use of (i) or given properties																									
	OR $ap^2 = a \cdot a^2 = a^3 \Rightarrow$ $(ap^2)^2 = a^6 = a^2$ $\Rightarrow \text{order } ap^2 = 4$	A1 5	For correct order with no incorrect working seen																									
(iii)	METHOD 1 $p^2 = a^2, ap^2 = a^3$	M2	For use of the given properties to simplify p^2 and ap^2																									
	$\Rightarrow \{e, a, p^2, ap^2\} = \{e, a, a^2, a^3\}$ which is a cyclic group	A1 A1 4	For obtaining a^2 and a^3 For justifying that the set is a group																									
	METHOD 2																											
	<table border="1"> <thead> <tr> <th></th> <th>e</th> <th>a</th> <th>p^2</th> <th>ap^2</th> </tr> </thead> <tbody> <tr> <th>e</th> <td>e</td> <td>a</td> <td>p^2</td> <td>ap^2</td> </tr> <tr> <th>a</th> <td>a</td> <td>p^2</td> <td>ap^2</td> <td>e</td> </tr> <tr> <th>p^2</th> <td>p^2</td> <td>ap^2</td> <td>e</td> <td>a</td> </tr> <tr> <th>ap^2</th> <td>ap^2</td> <td>e</td> <td>a</td> <td>p^2</td> </tr> </tbody> </table>		e	a	p^2	ap^2	e	e	a	p^2	ap^2	a	a	p^2	ap^2	e	p^2	p^2	ap^2	e	a	ap^2	ap^2	e	a	p^2	M1 A1	For attempting closure with all 9 non-trivial products seen For all 16 products correct
	e	a	p^2	ap^2																								
e	e	a	p^2	ap^2																								
a	a	p^2	ap^2	e																								
p^2	p^2	ap^2	e	a																								
ap^2	ap^2	e	a	p^2																								
	Completed table is a cyclic group	B2	For justifying that the set is a group																									
	METHOD 3																											
	<table border="1"> <thead> <tr> <th></th> <th>e</th> <th>a</th> <th>p^2</th> <th>ap^2</th> </tr> </thead> <tbody> <tr> <th>e</th> <td>e</td> <td>a</td> <td>p^2</td> <td>ap^2</td> </tr> <tr> <th>a</th> <td>a</td> <td>p^2</td> <td>ap^2</td> <td>e</td> </tr> <tr> <th>p^2</th> <td>p^2</td> <td>ap^2</td> <td>e</td> <td>a</td> </tr> <tr> <th>ap^2</th> <td>ap^2</td> <td>e</td> <td>a</td> <td>p^2</td> </tr> </tbody> </table>		e	a	p^2	ap^2	e	e	a	p^2	ap^2	a	a	p^2	ap^2	e	p^2	p^2	ap^2	e	a	ap^2	ap^2	e	a	p^2	M1 A1	For attempting closure with all 9 non-trivial products seen For all 16 products correct
	e	a	p^2	ap^2																								
e	e	a	p^2	ap^2																								
a	a	p^2	ap^2	e																								
p^2	p^2	ap^2	e	a																								
ap^2	ap^2	e	a	p^2																								
	Identity = e	B1	For stating identity																									
	Inverses exist since EITHER: e is in each row/column OR: p^2 is self-inverse; a, ap^2 form an inverse pair	B1	For justifying inverses ($e^{-1} = e$ may be assumed)																									

(iv) METHOD 1 e.g. $a \cdot ap = a^2 p = p^3$ $ap \cdot a = p$ commutative	M1	For attempting to find a non-commutative pair of elements, at least one involving a (may be embedded in a full or partial table)
	M1	For simplifying elements both ways round
	B1	For a correct pair of non-commutative elements
	A1	4 For stating Q non-commutative, with a clear argument
<hr/>		
METHOD 2 Assume commutativity, so (eg) $ap = pa$	M1	For setting up proof by contradiction
(i) \Rightarrow $p = ap \cdot a \Rightarrow p = pa \cdot a = pa^2 = pp^2 = p^3$	M1	For using (i) and/or given properties
But p and p^3 are distinct	B1	For obtaining and stating a contradiction
$\Rightarrow Q$ is non-commutative	A1	For stating Q non-commutative, with a clear argument
<hr/>		
15		

4728 Mechanics 1

1 i	$x^2 + (3x)^2 = 6^2$ $10x^2 = 36$ $x = 1.9(0)$ (1.8973..)	M1 A1 A1 [3]	Using Pythagoras, 2 squared terms May be implied Not surd form unless rationalised $(3\sqrt{10})/5$, $(6\sqrt{10})/10$
ii	$\tan\theta = 3x/x (= 3 \times 1.9/1.9) = 3$ $\theta = 71.6^\circ$ (71.565..)	M1 A2 [3]	Must target correct angle. Accept $\sin\theta = 3 \times 1.9/6$ or $\cos\theta = 1.9/6$ which give $\theta = 71.8^\circ$, $\theta = 71.5^\circ$ respectively, A1. SR $\theta = 71.6^\circ$ from $\tan\theta = 3x/x$ if x is incorrect; x used A1, no evidence of x used A2
2 i		B1 B1 [2]	Inverted V shape with straight lines. Starts at origin, ends on t -axis, or horizontal axis if no labelling evident
ii	$6 = 3v/2$ $v = 4 \text{ ms}^{-1}$	M1 A1 A1 [3]	Not awarded if special (right angled, isosceles) triangle assumed, or $s = (u+v)t/2$, or max v at specific t .
iii	T accn = $4/2.4$ or s accn = $16/(2 \times 2.4)$ T accn = $1 \frac{2}{3}$ s or s accn = $10/3$ Deceleration = $4/(3 - 1 \frac{2}{3})$ or $16/2(6-10/3)$ Deceleration = 3 ms^{-2}	M1* A1 D*M1 A1 [4]	Uses $t = v/a$ or $s = v^2/2a$. May be implied Accept $4/(3 - 1.67)$ or $16/2(6-3.33)$ Accept 3.01; award however $v = 4$ obtained in (ii). $a = -3$ gets A0.
3 i	$0.8g\sin 30$ 0.8×0.2 $0.8 \times 9.8\sin 30 - T = 0.8 \times 0.2$ $T = 3.76 \text{ N}$	B1 B1 M1 A1 [4]	Not for 3.92 stated without justification Or 0.16 Uses N2L // to slope, 3 non-zero terms, inc ma Not awarded if initial B1 withheld.
ii	$3.76 - F = 3 \times 0.2$ $F = 3.16$ $3.16 = \mu R \times 9.8$ $\mu = 0.107$ (0.10748)	M1 A1 A1 M1 A1 [5]	Uses N2L, B alone, 3 non-zero terms Needs <i>correct value</i> of T . May be implied. Uses $F = \mu R$ (Accept with $R = 3$, but not with $R = 0.8g(\cos 30)$, $F = 0.6$, $F = 3.76$, $F = f(\text{mass } P)$) Not 0.11, 0.108 (unless it comes from using $g = 9.81$ consistently through question.

4 i	$v^2 = 7^2 - 2 \times 9.8 \times 2.1$ $v = 2.8 \text{ ms}^{-1}$	M1 A1 A1 [3]	Uses $v^2 = u^2 - 2gs$. Accept $7^2 = u^2 + 2gs$
ii	$v = 0$ $0^2 = 7^2 - 2 \times 9.8s$ $s = 2.5 \text{ m}$	B1 M1 A1 [3]	Velocity = 0 at greatest height Uses $0 = u^2 - 2gs$. Accept $7^2 = 2 \times 9.8s$.
iii	$v = -5.7$ (or $t = 0.71$ oef to reach greatest height) $-5.7 = 7 - 9.8t$ or $5.7 = (0+) 9.8T$ $t = 1.3(0) \text{ s}$ (1.2959..)	B1 M1 A1 [3]	Allows for change of direction Uses $v = u +$ or $-gt$. Not 1.29 unless obtained from $g=9.81$ consistently
5 i	$0.5 \times 6 = 0.5v + m(v+1)$ $3 = 0.5v + mv + m$ $v(m + 0.5) = -m + 3$	M1 A1 A1 [3] AG	Uses CoLM. Includes g throughout MR-1
ii	Momentum before = +/- $(4m - 0.5 \times 2)$ $\pm (4m - 0.5 \times 2) = mv + 0.5(v+1)$ $4m - 0.5 \times 2 = mv + 0.5(v+1)$ $v(m+0.5) = 4m - 1.5$	B1 M1 A1 A1 [4]	Includes g throughout MR-1 Needs opposite directions in CoLM on "before" side only. RHS in format $am + b$ or $b + am$. Ignore values for a and b if quoted.
iii	$4m - 1.5 = -m + 3$ $5m = 4.5$ $m = 0.9 \text{ kg}$ $0.9 + v(0.9+0.5) = 3$ or $4 \times 0.9 - 1.5 = v(0.9+0.5)$ $v = (3-0.9)/(0.9+0.5) = 2.1/1.4$ $v = 1.5 \text{ ms}^{-1}$	M1 A1 M1 A1 [4] AG	Attempts to obtain eqn in 1 variable from answers in (i) and (ii) Ignore $m = -0.5$ if seen Substitutes for $m=0.9$ in any m, v equation obtained earlier.
6 ia	$\text{Perp} = 10\cos 20$ (= 9.3967 or 9.4) $// = 10\sin 20$ (= 3.4202)	B1 B1 [2]	Includes g , MR -1 in part (i). Accept $-ve$ values.
b	$\mu = 10\sin 20 / 10\cos 20 = \tan 20$ (= 3.42/9.4) $\mu = 0.364$ (0.36397..)	M1 A1 [2] AG	Must use $ F = \mu R $ Accept after inclusion of g twice
ii	<i>No misread, and resolving of 10 and T required</i> $R = 10\cos 20 + T\cos 45$ $F = T\cos 45 - 10\sin 20$ or $T\cos 45 = \mu R + 10\sin 20$ $T\cos 45 - 3.42 = 0.364(9.4 + T\cos 45)$ $0.707T - 3.42 = 3.42 + 0.257T$ $0.45T = 6.84$ $T = 15.2 \text{ N}$ (15.209..)	M1* A1 M1* A1 D*M1 A1 A1 [7]	3 term equation perp plane, 2 unknowns $9.4 + 0.707T$ (accept $9.4 + .71T$) 3 term equation // plane, 2 unknowns $0.707T - 3.42$ (accept $0.71T - 3.4$) Substitutes for F and R in $F=0.364R$ <i>Award final A1 only for $T = 149 \text{ N}$ after using $10g$ for weight</i>

7 i	$a = dv/dt$ $a = 6 - 2t \text{ ms}^{-2}$	M1 A1 [2]	Differentiation attempt. Answer $6-t$ implies division by t
ii	$s = \int v dt$ $s = \int 6t - t^2 dt$ $s = 3t^2 - t^3/3 (+c)$ $t = 0, v = 0, c = 0$ $t = 3, s = 3 \times 3^2 - 3^3/3$ $s = 18 \text{ m}$	AG M1* A1 B1 D*M1 A1 [5]	Integration attempt on v Award if limits 0,3 used Requires earlier integration Does not require B1 to be earned.
iii	Distance remaining $(= 100 - 18) = 82$ Total time $= 3 + 82/9$ $T = 12.1 \text{ s}$ (12 1/9)	B1 M1 A1 [3]	Numerator not 100 Not 109/9
iv	Distance before slows $= 18 + (22 - 3) \times 9$ Distance while decelerating $= 200 - 189 = 11$ $11 = 9t - 0.3t^2$ or $11 = (9 + 8.23)t/2$ or $8.23 = 9 - 0.6t$ $t = 1.28$ (1.2765..., accept 1.3) $T = 23.3 \text{ s}$ (23.276..)	M1* A1 D*M1 A1 D*M1 A1 A1 [7]	(=189 m) Two sub-regions considered Accept 10.99. 10.9 penalise -1PA. Uses $s = ut - 0.5 \times 0.6t^2$, or $v^2 = u^2 - 2 \times 0.6s$ with $s = (u+v)t/2$ or $v = u + at$ Finds t . (If QE, it must have 3 terms and smaller positive root chosen.)

4729 Mechanics 2

1 (i)	$\frac{1}{2} \times 75 \times 12^2$ or $\frac{1}{2} \times 75 \times 3^2$ (either KE) $75 \times 9.8 \times 40$ (PE) $R \times 180$ (change in energy = 24337) $\frac{1}{2} \times 75 \times 12^2 = \frac{1}{2} \times 75 \times 3^2 + 75 \times 9.8 \times 40 - R \times 180$ $R = 135 \text{ N}$	B1 B1 B1 M1 A1 5	$M1 \quad 12^2 = 3^2 + 2a \times 180$ $A1 \quad a = 0.375 \quad (3/8)$ $M1 \quad 75 \times 9.8 \times \sin \theta - R = 75a$ $A1 \quad R = 135$ (max 4 for no energy)	5
2 (i)	$R = F = P/v = 44\,000/v = 1400$ $v = 31.4 \text{ m s}^{-1}$	M1 A1 2		
(ii)	$44\,000/v = 1400 + 1100 \times 9.8 \times 0.05$ $v = 22.7 \text{ m s}^{-1}$	M1 A1 A1 3	must have g	
(iii)	$22\,000/10 + 1100 \times 9.8 \times 0.05 - 1400$ $= 1100a$ $a = 1.22 \text{ m s}^{-2}$	M1 A1 A1 3		8
3 (i)	$\cos \theta = 5/13$ or $\sin \theta = 12/13$ or $\theta = 67.4^\circ$ $0.5 \times F \sin \theta = 70 \times 1.4 + 50 \times 2.8$ $F = 516 \text{ N}$	B1 M1 A1 A1 4	any one of these moments about A (ok without 70) $0.5 \sin \theta = 0.4615$ SR 1 for 303 (omission of beam)	
(ii)	$F \sin \theta = 120 + Y$ (resolving vertically) $Y = 356$ ✓ their $F \times 12/13 - 120$ $X = F \cos \theta$ (resolving horizontally) $X = 198$ ✓ their $F \times 5/13$ Force = $\sqrt{(356^2 + 198^2)}$ 407 or 408 N	M1 A1 ✓ M1 A1 ✓ M1 A1 6	M1/A1 for moments (B) $Y \times 2.8 + 1.4 \times 70 = 2.3 \times 516 \times 12/13$ (C) $0.5 \times Y = 0.9 \times 70 + 2.3 \times 50$ (D) $1.2X = 1.4 \times 70 + 2.8 \times 50$	10
4 (i)	$T = 0.4 \times 0.6 \times 2^2$ $T = 0.96 \text{ N}$	M1 A1 2		
(ii)	$S - T$ $S - T = 0.1 \times 0.3 \times 2^2$ $S = 1.08$	B1 M1 A1 A1 4	may be implied	
(iii)	$v = r\omega$ $v_P = 0.6$ $v_B = 1.2$ $\frac{1}{2} \times 0.1 \times 0.6^2 + \frac{1}{2} \times 0.4 \times 1.2^2$ 0.306	M1 A1 A1 M1 A1 5	(0.018 + 0.288) separate speeds	11

5 (i)	$\bar{d} = (2 \times 6 \sin \pi/4) / 3\pi/4$ $\bar{d} = 3.60$	M1 A1 2	must be correct formula with rads AG	
(ii)	$\bar{d} \cos 45^\circ = "2.55"$ $5\bar{x} = 3 \times 3 + 2 \times "2.55"$ $\bar{x} = 2.82$ $5\bar{y} = 3 \times 6 + 2 \times (12 + "2.55")$ $\bar{y} = 9.42$	B1 M1 A1 A1 M1 A1 A1 7	may be implied moments must not have areas 2kg/3kg misread (swap) gives (2.73, 11.13) $\theta = 21.7^\circ$ (MR - 2) (max 7 for (ii) + (iii)) SR -1 for \bar{x} , \bar{y} swap	
(iii)	$\tan \theta = 2.82/8.58$ $\theta = 18.2^\circ$	M1 A1 2	M0 for their \bar{x} / \bar{y} their $\bar{x} / (18 - \bar{y})$	11
6 (i)	$I = 0.9 = 6 \times 0.2 - v \times 0.2$ $v = 1.5$	M1 A1 A1 3	needs to be mass 0.2	
(ii)	$0.6 = (c - b) / 6$ $6 \times 0.2 = 0.2b + 0.1c$ $b = 2.8$ $0.4 \times 5 + 0.2 \times 1.5 = 0.4a + 0.2 \times 6 \quad \text{or}$ $I = 0.9 = -0.4a - 0.4 \times 5$ $a = 2.75$ $2.75 < 2.8$ no further collision	M1 A1 M1 A1 A1 M1 A1 M1 A1 10	restitution (allow 1.5 for M1) momentum (allow 1.5 for M1) 1st collision (needs their 1.5 for M1) compare v 's of A and B (calculated)	13
7(i)	$9 = 17 \cos 25^\circ \times t$ $t = 0.584 \quad (\text{or } 9/17 \cos 25^\circ)$ $d = 17 \sin 25^\circ \times 0.584 + \frac{1}{2} \times 9.8 \times 0.584^2 \quad (d = ht \text{ lost } (5.87))$ $h = 2.13$	M1 A1 M1 A1 A1 5	B1 $y = x \tan \theta - 4.9x^2/v^2 \cos^2 \theta$ M1/A1 $y = 9 \tan(-25^\circ) - 4.9 \times 9^2 / 17^2 \cos^2 25^\circ$ A1 $y = -5.87$ 2.13	
(ii)	$v_h = 17 \cos 25^\circ \quad (15.4)$ $v_v = 17 \sin 25^\circ + 9.8 \times 0.584$ $v_v^2 = (17 \sin 25^\circ)^2 + 2 \times 9.8 \times 5.87$ $v_v = 12.9$ $\tan \theta = 12.9/15.4$ $\theta = 40.0^\circ \text{ below horizontal}$	B1 M1 A1 M1 A1 5	M1/A1 $dy/dx = \tan \theta - 9.8x/v^2 \cos^2 \theta$ A1 $dy/dx = -0.838$ M1 $\tan^{-1}(-0.838)$ or 50.0° to vertical	
(iii)	$\text{speed} = \sqrt{(12.9^2 + 15.4^2)}$ $\frac{1}{2}mv^2 = \frac{1}{2}m \times 20.1^2 \times 0.7$ $v = 16.8 \text{ m s}^{-1}$	M1 A1 M1 A1 4	(20.1) NB 0.3 instead of 0.7 gives 11.0 (M0)	14

4730 Mechanics 3

1 i	Horiz. comp. of vel. after impact is 4ms^{-1} Vert. comp. of vel. after impact is $\sqrt{5^2 - 4^2} = 3\text{ms}^{-1}$ Coefficient of restitution is 0.5	B1 B1 B1 [3]	May be implied AG From $e = 3/6$
ii	Direction is vertically upwards Change of velocity is $3 - (-6)$ Impulse has magnitude 2.7Ns	B1 M1 A1 [3]	From $m(\Delta v) = 0.3 \times 9$
2 i	Horizontal component is 14N $80 \times 1.5 = 14 \times 1.5 + 3Y$ or $3(80 - Y) = 80 \times 1.5 + 14 \times 1.5$ or $1.5(80 - Y) = 14 \times 0.75 + 14 \times 0.75 + 1.5Y$ Vertical component is 33N upwards	B1 M1 A1 A1 [4]	For taking moments for AB about A or B or the midpoint of AB AG
ii	Horizontal component at C is 14N [Vertical component at C is $(\pm)\sqrt{50^2 - 14^2}$] $[W = (\pm)48 - 33]$ Weight is 15N	B1 M1 DM1 A1 [4]	May be implied for using $R^2 = H^2 + V^2$ For resolving forces at C vertically
3 i	$4 \times 3 \cos 60^\circ - 2 \times 3 \cos 60^\circ = 2b$ $b = 1.5$ j component of vel. of $B = (-)3 \sin 60^\circ$ $[v^2 = b^2 + (-3 \sin 60^\circ)^2]$ Speed (3ms^{-1}) is unchanged [Angle with l.o.c. = $\tan^{-1}(3 \sin 60^\circ / 1.5)$] Angle is 60° .	M1 A1 A1 B1ft M1 A1ft M1 A1ft [8]	For using the p.c.mmtm parallel to l.o.c. ft consistent sin/cos mix For using $v^2 = b^2 + v_y^2$ AG ft - allow same answer following consistent sin/cos mix. For using angle = $\tan^{-1}(\pm v_y/v_x)$ ft consistent sin/cos mix
ii	$[e(3 \cos 60^\circ + 3 \cos 60^\circ) = 1.5]$ Coefficient is 0.5	M1 A1ft [2]	For using NEL ft - allow same answer following consistent sin/cos mix throughout.

4 i	$F - 0.25v^2 = 120v(\text{dv}/\text{dx})$ $F = 8000/v$ $[32000 - v^3 = 480v^2(\text{dv}/\text{dx})]$ $\frac{480v^2}{v^3 - 32000} \frac{\text{d}v}{\text{d}x} = -1$	M1 A1 B1 M1 A1 [5]	For using Newton's second law with $a = v(\text{dv}/\text{dx})$ For substituting for F and multiplying throughout by $4v$ (or equivalent) AG
ii	$\int \frac{480v^2}{v^3 - 32000} \text{d}v = -\int \text{d}x$ $160 \ln(v^3 - 32000) = -x (+A)$ $160 \ln(v^3 - 32000) = -x + 160 \ln 32000$ or $160 \ln(v^3 - 32000) - 160 \ln 32000 = -500$ $(v^3 - 32000)/32000 = e^{-x/160}$ Speed of m/c is 32.2ms^{-1}	M1 A1 M1 A1ft B1ft B1 [6]	For separating variables and integrating For using $v(0) = 40$ or $[160 \ln(v^3 - 32000)]^v_{40} = [-x]^{500}_0$ ft where factor 160 is incorrect but +ve, Implied by $(v^3 - 32000)/32000 = e^{-3.125}$ (or = 0.0439 ..). ft where factor 160 is incorrect but +ve, or for an incorrect non-zero value of A
5 i	$x_{\text{max}} = \sqrt{1.5^2 + 2^2} - 1.5 (= 1)$ $[T_{\text{max}} = 18 \times 1/1.5]$ Maximum tension is 12N	B1 M1 A1 [3]	For using $T = \lambda x/L$
	(a) Gain in EE = $2[18(1^2 - 0.2^2)]/(2 \times 1.5)$ (11.52) Loss in GPE = $2.8mg$ (27.44m) ii $[2.8m \times 9.8 = 11.52]$ $m = 0.42$ (b) $\frac{1}{2}mv^2 = mg(0.8) + 2 \times 18 \times 0.2^2 / (2 \times 1.5)$ or $\frac{1}{2}mv^2 = 2 \times 18 \times 1^2 / (2 \times 1.5) - mg(2)$ Speed at M is 4.24ms^{-1}	M1 A1 B1 M1 A1 A1ft A1ft [5] M1 A1ft A1ft [3]	For using $EE = \lambda x^2/2L$ May be scored with correct EE terms in expressions for total energy on release and total energy at lowest point May be scored with correct GPE terms in expressions for total energy on release and total energy at lowest point For using the p.c.energy AG For using the p.c.energy KE, PE & EE must all be represented ft only when just one string is considered throughout in evaluating EE ft only for answer 4.10 following consideration of only one string

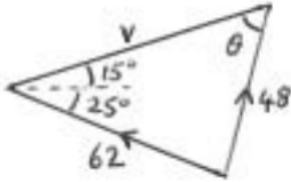
6 i	$[-mg \sin \theta = m L(d^2 \theta / dt^2)]$ $d^2 \theta / dt^2 = -(g/L) \sin \theta$	M1 A1 [2]	For using Newton's second law tangentially with $a = Ld^2 \theta / dt^2$ AG
ii	$[d^2 \theta / dt^2 = -(g/L) \theta]$ $d^2 \theta / dt^2 = -(g/L) \theta \rightarrow \text{motion is SH}$	M1 A1 [2]	For using $\sin \theta \approx \theta$ because θ is small ($\theta_{\max} = 0.05$) AG
iii	$[4\pi/7 = 2\pi/\sqrt{9.8/L}]$ $L = 0.8$	M1 A1 [2]	For using $T = 2\pi/n$ where n^2 is coefficient of θ
iv	$[\theta = 0.05 \cos 3.5 \times 0.7]$ $\theta = -0.0385$ $t = 1.10 \text{ (accept 1.1 or 1.09)}$	M1 A1ft M1 A1ft [4]	For using $\theta = \theta_0 \cos nt$ { $\theta = \theta_0 \sin nt$ not accepted unless the t is reconciled with the t as defined in the question } ft incorrect L { $\theta = 0.05 \cos [4.9/(5L)^{1/2}]$ } For attempting to find $3.5t$ ($\pi < 3.5t < 1.5\pi$) for which $0.05 \cos 3.5t =$ answer found for θ or for using $3.5(t_1 + t_2) = 2\pi$ ft incorrect L { $t = [2\pi(5L)^{1/2}]/7 - 0.7$ }
v	$\theta^2 = 3.5^2(0.05^2 - (-0.0385)^2) \text{ or}$ $\theta^2 = -3.5 \times 0.05 \sin(3.5 \times 0.7) \quad (\theta^2 = -0.1116..)$ $\text{Speed is } 0.0893 \text{ms}^{-1}$ $\text{(Accept answers correct to 2 s.f.)}$	M1 A1ft A1ft [3]	For using $\theta^2 = n^2(\theta_0^2 - \theta^2)$ or $\theta^2 = -n \theta_0 \sin nt$ {also allow $\theta^2 = n \theta_0 \cos nt$ if $\theta = \theta_0 \sin nt$ has been used previously} ft incorrect θ with or without 3.5 represented by $(g/L)^{1/2}$ using incorrect L in (iii) or for $\theta^2 = 3.5 \times 0.05 \cos(3.5 \times 0.7)$ following previous use of $\theta = \theta_0 \sin nt$ ft incorrect L ($L \times 0.089287/0.8$ with $n = 3.5$ used or from $ 0.35 \sin \{4.9/[5L]^{1/2}\}/[5L]^{1/2} $) SR for candidates who use θ^2 as v . (Max 1/3) For $v = \pm 0.112$ B1

7 i	Gain in PE = $mga(1 - \cos \theta)$ $[\frac{1}{2} mu^2 - \frac{1}{2} mv^2 = mga(1 - \cos \theta)]$	B1 M1	For using KE loss = PE gain
	$v^2 = u^2 - 2ga(1 - \cos \theta)$ $[R - mg \cos \theta = m(\text{accel.})]$ $R = mv^2/a + mg \cos \theta$ $[R = m \{ u^2 - 2ga(1 - \cos \theta) \} / a + mg \cos \theta]$ $R = mu^2/a + mg(3 \cos \theta - 2)$	A1 M1 A1 M1 A1 [7]	For using Newton's second law radially For substituting for v^2 AG
ii	$[0 = mu^2/a - 5mg]$ $u^2 = 5ag$ $[v^2 = 5ag - 4ag]$ Least value of v^2 is ag	M1 A1 M1 A1 [4]	For substituting $R = 0$ and $\theta = 180^\circ$ For substituting for $u^2 (= 5ag)$ and $\theta = 180^\circ$ in v^2 (expression found in (i)) { but M0 if $v = 0$ has been used to find u^2 } AG
iii	$[0 = u^2 - 2ga(1 - \frac{\sqrt{3}}{2})]$ $u^2 = ag(2 - \sqrt{3})$	M1 A1 [2]	For substituting $v^2 = 0$ and $\theta = \pi/6$ in v^2 (expression found in (i)) Accept $u^2 = 2ag(1 - \cos \pi/6)$

4731 Mechanics 4

1 (i)	Using $\omega_2^2 = \omega_1^2 + 2\alpha\theta$, $67^2 = 83^2 + 2\alpha \times 1000$ $\alpha = -1.2$ Angular deceleration is 1.2 rad s^{-2}	M1 A1 [2]	
1 (ii)	Using $\theta = \omega_1 t + \frac{1}{2}\alpha t^2$, $400 = 83t - 0.6t^2$ $t = 5 \text{ or } 133\frac{1}{3}$ Time taken is 5 s	M1 A1 ft M1 A1 [4]	Solving to obtain a value of t
Alternative for (ii) $\omega_2^2 = 83^2 - 2 \times 1.2 \times 400$ M1A1 ft $\omega_2 = 77$ $77 = 83 - 1.2t$ M1 $t = 5$ A1		(M0 if $\omega = 67$ is used in (ii))	
2	$\text{Volume } V = \int_a^{2a} \pi y^2 dx = \int_a^{2a} \pi \frac{a^6}{x^4} dx$ $= \pi \left[-\frac{a^6}{3x^3} \right]_a^{2a} = \frac{7}{24} \pi a^3$ $V \bar{x} = \int_a^{2a} \pi xy^2 dx$ $= \int_a^{2a} \pi \frac{a^6}{x^3} dx$ $= \pi \left[-\frac{a^6}{2x^2} \right]_a^{2a} = \frac{3}{8} \pi a^4$ $\bar{x} = \frac{\frac{3}{8} \pi a^4}{\frac{7}{24} \pi a^3}$ $= \frac{9a}{7}$	M1 A1 M1 A1 A1 M1 A1 [7]	π may be omitted throughout For integrating x^{-4} to obtain $-\frac{1}{3}x^{-3}$ for $\int xy^2 dx$ Correct integral form (including limits) For integrating x^{-3} to obtain $-\frac{1}{2}x^{-2}$ Dependent on previous M1M1

3 (i)	$I = \frac{1}{2}(4m)(2a)^2 + (4m)a^2$ $+ m(3a)^2$ $= 21ma^2$	M1 A1 B1 A1 [4]	Applying parallel axes rule
(ii)	<p>From P, $\bar{x} = \frac{(4m)a + m(3a)}{5m} (= \frac{7a}{5})$</p> <p>Period is $2\pi \sqrt{\frac{21ma^2}{5mg(\frac{7}{5}a)}}$</p> $= 2\pi \sqrt{\frac{3a}{g}}$ <hr/> <p><i>Alternative for (ii)</i></p> $-4mga \sin \theta - mg(3a) \sin \theta = (21ma^2)\theta^2$ <p>Period is $2\pi \sqrt{\frac{21ma^2}{7mga}} = 2\pi \sqrt{\frac{3a}{g}}$</p>	M1 M1 A1 ft A1 [4]	<p>Correct formula $2\pi \sqrt{\frac{I}{mgh}}$ seen <i>or</i> using $L = I\theta^2$ and period $2\pi/\omega$</p> <hr/> <p>Using $L = I\theta^2$ with three terms Using period $2\pi/\omega$</p>

4 (i)	 $\frac{\sin \theta}{62} = \frac{\sin 40}{48}$ $\theta = 56.1^\circ \text{ or } 123.9^\circ$ Bearings are 018.9° and 311.1°	M1 M1 A1 A1A 1 [5]	Velocity triangle One value sufficient Accept 19° and 311°
(ii)	Shorter time when $\theta = 56.1^\circ$ $\frac{v}{\sin 83.87} = \frac{48}{\sin 40}$ Relative speed is $v = 74.25$ Time to intercept is $\frac{3750}{74.25}$ $= 50.5 \text{ s}$	B1 ft M1 M1 A1 [4]	Or $v^2 = 62^2 + 48^2 - 2 \times 62 \times 48 \cos 83.87$ <i>Dependent on previous M1</i>
	<i>Alternative for (i) and (ii)</i> $\begin{pmatrix} 48 \sin \phi \\ 48 \cos \phi \end{pmatrix} t = \begin{pmatrix} 3750 \sin 75 \\ 3750 \cos 75 \end{pmatrix} + \begin{pmatrix} 62 \sin 295 \\ 62 \cos 295 \end{pmatrix} t$ $3.732 \cos \phi - \sin \phi = 3.208$ $\phi = 18.9^\circ \text{ and } 311.1^\circ$ $t = 50.5$	M1 M1 A1 M1 M1 A1A1 B1 ft A1	component eqns (displacement or velocity) obtaining eqn in ϕ or t or v ($=3750/t$) correct simplified equation or $t^2 - 231.3t + 9131.5 = 0$ [$t = 50.5, 180.8$] or $v^2 - 94.99v + 1540 = 0$ [$v = 74.25, 20.74$] solving to obtain a value of ϕ solving to obtain a value of t <i>(max A1 if any extra values given)</i> appropriate selection for shorter time

5 (i)	<p>Area is $\int_0^2 (8-x^3) dx = \left[8x - \frac{1}{4}x^4 \right]_0^2 = 12$</p> <p>Mass per m^2 is $\rho = \frac{63}{12} = 5.25$</p> <p>$I_y = \sum (\rho y \delta x) x^2 = \rho \int x^2 y dx$</p> <p>$= \rho \int_0^2 (8x^2 - x^5) dx$</p> <p>$= \rho \left[\frac{8}{3}x^3 - \frac{1}{6}x^6 \right]_0^2 = \frac{32}{3}\rho$</p> <p>$= \frac{32}{3} \times \frac{63}{12} = 56 \text{ kg m}^2$</p>	<p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>AG</p> <p>[6]</p>	<p>for $\int x^2 y dx$ or $\int x^3 dy$</p> <p>or $\frac{1}{3}\rho \int_0^8 (8-y) dy$</p> <p>for $\frac{32}{3}$</p>
(ii)	<p>Anticlockwise moment is $800 - 63 \times 9.8 \times \frac{4}{5}$</p> <p>$= 306.08 \text{ N m} > 0$</p> <p>so it will rotate anticlockwise</p>	<p>M1</p> <p>A1</p> <p>[2]</p>	<p>Full explanation is required; (anti)clockwise should be mentioned before the conclusion</p>
(iii)	<p>$I = I_x + I_y = 1036.8 + 56 (=1092.8)$</p> <p>WD by couple is $800 \times \frac{1}{2}\pi$</p> <p>Change in PE is $63 \times 9.8 \times \left(\frac{24}{7} - \frac{4}{5} \right)$</p> <p>$800 \times \frac{1}{2}\pi = \frac{1}{2} I \omega^2 - 63 \times 9.8 \times \left(\frac{24}{7} - \frac{4}{5} \right)$</p> <p>$1256.04 = 546.4 \omega^2 - 1622.88$</p> <p>$\omega = 2.30 \text{ rad s}^{-1}$</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[6]</p>	<p>Equation involving WD, KE and PE <i>May have an incorrect value for I; other terms and signs are cao</i></p>

6 (i)	GPE is $mg(a \sin 2\theta)$ $AB = 2a \cos \theta$ or $AB^2 = a^2 + a^2 - 2a^2 \cos(\pi - 2\theta)$ EPE is $\frac{\sqrt{3}mg}{2a}(2a \cos \theta)^2$ $= \sqrt{3}mga(1 + \cos 2\theta)$ Total PE is $V = \sqrt{3}mga(1 + \cos 2\theta) + mga \sin 2\theta$ $= mga(\sqrt{3} + \sqrt{3} \cos 2\theta + \sin 2\theta)$	B1 B1 M1 A1 AG [4]	Or $mg(2a \cos \theta \sin \theta)$ Any correct form Expressing EPE and GPE in terms of $\cos 2\theta$ and $\sin 2\theta$
(ii)	$\frac{dV}{d\theta} = mga(-2\sqrt{3} \sin 2\theta + 2 \cos 2\theta)$ $= 0$ when $2\sqrt{3} \sin 2\theta = 2 \cos 2\theta$ $\tan 2\theta = \frac{1}{\sqrt{3}}$ $\theta = \frac{\pi}{12}, -\frac{5\pi}{12}$	B1 M1 M1 A1A1 [5]	(B0 for $\frac{dV}{d\theta} = -2\sqrt{3} \sin 2\theta + 2 \cos 2\theta$) Solving to obtain a value of θ Accept $0.262, -1.31$ or $15^\circ, -75^\circ$
(iii)	$\frac{d^2V}{d\theta^2} = mga(-4\sqrt{3} \cos 2\theta - 4 \sin 2\theta)$ When $\theta = \frac{\pi}{12}, \frac{d^2V}{d\theta^2} = -8mga < 0$ so this position is unstable When $\theta = -\frac{5\pi}{12}, \frac{d^2V}{d\theta^2} = 8mga > 0$ so this position is stable	B1ft M1 A1 A1 [4]	Determining the sign of V'' or M2 for alternative method for max / min

7 (i)	Initially $\cos\theta = \frac{0.6}{1.5} = 0.4$ $\frac{1}{2} \times 4.9 \omega^2 = 6 \times 9.8(0.5 \times 0.4 - 0.5 \cos\theta)$ $\omega^2 = 12(0.4 - \cos\theta)$ $\omega^2 = 4.8 - 12 \cos\theta$	M1 A1 A1 AG [3]	Equation involving KE and PE
(ii)	$6 \times 9.8 \times 0.5 \sin\theta = 4.9\alpha$ $\alpha = 6 \sin\theta \text{ (rads}^{-2}\text{)}$	M1 A1 [2]	or $2\omega \frac{d\omega}{d\theta} = 12 \sin\theta$ or $2\omega \frac{d\omega}{dt} = 12 \sin\theta \frac{d\theta}{dt}$
(iii)	$6 \times 9.8 \cos\theta - F = 6 \times 0.5 \omega^2$ $58.8 \cos\theta - F = 14.4 - 36 \cos\theta$ $F = 94.8 \cos\theta - 14.4$ $6 \times 9.8 \sin\theta - R = 6 \times 0.5 \alpha$ $58.8 \sin\theta - R = 18 \sin\theta$ $R = 40.8 \sin\theta$	M1 M1 A1 AG M1 M1 A1 [6]	for radial acceleration $r\omega^2$ radial equation of motion <i>Dependent on previous M1</i> for transverse acceleration $r\alpha$ transverse equation of motion <i>Dependent on previous M1</i>
(iv)	If B reaches the ground, $\cos\theta = -0.4$ $F = -52.32$ $\sin\theta = \sqrt{0.84}$ [$\theta = 1.982$ or 113.6°] $R = 37.39$ Since $\frac{52.32}{37.39} = 1.40 > 0.9$, this is not possible	M1 A1 M1 A1 [4]	<i>Allow M1A0 if $\cos\theta = +0.4$ is used</i> Obtaining a value for R Or $\mu R = 33.65$, and $52.32 > 33.65$
	<i>Alternative for (iv)</i> Slips when $F = -0.9R$ $94.8 \cos\theta - 14.4 = -36.72 \sin\theta$ M1 $\theta = 1.798$ [103.0°] A1 B reaches the ground when $\cos\theta = -0.4$ M1 $\theta = 1.982$ [113.6°] so it slips before this A1		<i>Allow M1A0 if $F = +0.9R$ is used</i> <i>Allow M1A0 if $\cos\theta = +0.4$ is used</i>

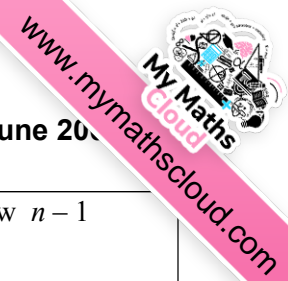
4732 Probability & Statistics 1

1			Q1: if consistent "0.8" incorrect or $1/8, 7/8$ or 0.02 allow M marks in ii, iii & 1 st M1 in i or implied by use of tables or 8C_3 or $0.2^a \times 0.8^b$ ($a+b=8$)
i	Binomial stated $0.9437 - 0.7969$ or ${}^8C_3 \times 0.2^3 \times 0.8^5$ $= 0.147$ (3 sfs)	M1 M1 A1 3	
ii	$1 - 0.7969$ $= 0.203$ (3 sf)	M1 A1 2	allow $1 - 0.9437$ or 0.056(3) or equiv using formula
iii	8×0.2 oe 1.6	M1 A1 2	$8 \times 0.2 = 2$ M1A0 $1.6 \div 8$ or $1/1.6$ M0A0
Total		7	
2	first two d' s = ± 1 Σd^2 attempted (= 2) $1 - \frac{6 \times "2"}{7(7^2 - 1)}$ $= 27/28$ or 0.964 (3 sfs)	B1 M1 M1dep A1	S_{xx} or $S_{yy} = 28$ B1 $S_{xy} = 27$ B1 $S_{xy} / \sqrt{(S_{xx}S_{yy})}$ M1 dep B1 1234567 & 1276543 (ans $2/7$): MR, lose A1
Total		4	
3 i	x independent or controlled or changed Value of y was measured for each x x not dependent	B1 1	Allow Water affects yield, or yield is dependent or yield not control water supply Not just y is dependent Not x goes up in equal intervals Not x is fixed
ii	(line given by) minimum sum of sqs	B1 B1 2	B1 for "minimum" or "least squares" with inadequate or no explanation
iii	$S_{xx} = 17.5$ or 2.92 $S_{yy} = 41.3$ or 6.89 $S_{xy} = 25$ or 4.17 $r = \frac{S_{xy}}{\sqrt{(S_{xx}S_{yy})}}$ $= 0.930$ (3 sf)	B1 M1 A1 3	or $91 - 21^{2/6}$ or $394 - 46^{2/6}$ B1 for any one or $186 - 21 \times 46/6$ dep B1 0.929 or 0.93 with or without wking B1M1A0 SC incorrect n : max B1M1A0
iv	Near 1 or lg, high, strong, good corr'n or relnshp oe Close to st line or line good fit	B1ft B1 2	$ r $ small: allow little (or no) corr'n oe Not line accurate. Not fits trend
Total		8	

4			Q4: if consistent "0.7" incorrect or $\frac{1}{3}$, $\frac{2}{3}$ or 0.03 allow M marks in ii, iii & 1 st M1 in i
i	Geo stated $0.7^3 \times 0.3$ alone $\frac{1029}{10000}$ or 0.103 (3 sf)	M1 M1 A1 3	or implied by $q^n \times p$ alone ($n > 1$) $0.7^3 - 0.7^4$
ii	0.7^4 alone $= \frac{2401}{10000}$ or 0.240 (3 sf)	M1 A1 2	$1 - (0.3 + 0.7 \times 0.3 + 0.7^2 \times 0.3 + 0.7^3 \times 0.3)$ NB $1 - 0.7^4$: M0
iii	$1 - 0.7^5$ $= 0.832$ (3 sfs)	M2 A1 3	or $0.3 + 0.7 \times 0.3 + \dots + 0.7^4 \times 0.3$ M2 M1 for one term extra or omitted or wrong or for 1- (above) M1 for $1 - 0.7^6$ or 0.7^5 NB Beware: $1 - 0.7^6 = 0.882$
		8	
5i	$\frac{25}{10}$ $= 2.5$	M1 A1 2	Allow $\frac{25}{(9 \text{ to } 10)}$ or 2.78: M1
ii	(19.5, 25) (9.5, 0)	B1 B1 2	Allow (24.5, 47) Both reversed: SC B1 If three given, ignore (24.5, 47)
iii	Don't know exact or specific values of x (or min or max or quartiles or median or whiskers). oe Can only estimate (min or max or quartiles or median or whiskers) oe Can't work out (.....) oe Data is grouped oe	B1 1	Exact data not known Allow because data is rounded
Total		5	

6i	$\Sigma x \div 11$ 70 Σx^2 attempted $\sqrt{\frac{\Sigma x^2}{11} - \bar{x}^2} = \sqrt{(54210/11 - 70^2)}$ or $\sqrt{28.18}$ or 5.309 (= 5.31) AG	M1 A1 M1 A1 4	≥ 5 terms, or $\Sigma(x - \bar{x})^2$ or $\sqrt{\frac{\Sigma(x - \bar{x})^2}{11}} = \sqrt{310/11}$ or $\sqrt{28.18}$ ie correct substn or result If $\times \frac{11}{10}$: M1A1M1A0
ii	Attempt arrange in order med = 67 74 and 66 IQR = 8	M1 A1 M1 A1 4	or $(72.5 - 76.5) - (65.5 - 66.5)$ incl must be from 74 – 66
iii	no (or fewer) extremes this year oe sd takes account of all values sd affected by extremes less spread tho' middle 50% same less spread tho' 3 rd & 9 th same or same gap	B1 1	iii, iv & v: ignore extras fewer high &/or low scores highest score(s) less than last year Not less spread or more consistent Not range less
iv	sd measures spread or variation or consistency oe	B1 1	sd less means spread is less oe or marks are closer together oe
v	more consistent, more similar, closer together, nearer to mean less spread	B1 1	allow less variance Not range less Not highest & lowest closer
Total		11	
7i	8C_3 $= 56$	M1 A1 2	
ii	7C_2 or or ${}^7P_2 / {}^8P_3$ $\div ({}^8C_3$ or "56") only $= \frac{3}{8}$	$\frac{1}{8}$ not from incorrect $\times 3$ only or $\frac{1}{8} + \frac{7}{8} \times \frac{1}{7} + \frac{7}{8} \times \frac{6}{7} \times \frac{1}{7}$	${}^8C_1 + {}^7C_1 + {}^6C_1$ or 21 or $8 \times 7 \times 6$ or $\frac{1}{8} \times \frac{1}{7} \times \frac{1}{6}$ $\frac{7}{8} \times \frac{6}{7} \times \frac{5}{6}$ 1 – prod 3 probs indep, dep ans < 1
iii	8P_3 or $8 \times 7 \times 6$ or ${}^8C_1 \times {}^7C_1 \times {}^6C_1$ or 336 $1 \div {}^8P_3$ only $= \frac{1}{336}$ or 0.00298 (3 sf)	M1 M1 A1 3	$\frac{1}{8} \times \frac{1}{7} \times \frac{1}{6}$ only M2 If \times or \div : M1 $(\frac{1}{8})^3$ M1
Total		8	

8ia	$\frac{18}{19}$ or $\frac{1}{19}$ seen $\frac{17}{18}$ or $\frac{1}{18}$ seen structure correct ie 6 branches all correct incl. probs and W & R	B1 B1 B1 B1 4	regardless of probs & labels (or 14 branches with correct 0s & 1s)
b	$\frac{1}{20} + \frac{19}{20} \times \frac{1}{19} + \frac{19}{20} \times \frac{18}{19} \times \frac{1}{18}$ $= \frac{3}{20}$	M2 A1 3	M1 any 2 correct terms added $\frac{19}{20} \times \frac{18}{19} \times \frac{17}{18}$ $1 - \frac{19}{20} \times \frac{18}{19} \times \frac{17}{18}$
ia	$\frac{19}{20} \times \frac{18}{19}$ $= \frac{9}{10}$ oe	M1 A1 2	$\frac{19}{20} \times \frac{18}{19} \times \frac{1}{18} + \frac{19}{20} \times \frac{18}{19} \times \frac{17}{18}$ or $\frac{1}{20} + \frac{17}{20}$
b	$(P(X=1) = \frac{1}{20})$ $\frac{19}{20} \times \frac{1}{19}$ $= \frac{1}{20}$ Σxp $= \frac{57}{20}$ or 2.85	M1 A1 M1 A1 4	or $1 - (\frac{1}{20} + \frac{9}{10})$ or 2 probs of $\frac{1}{20}$ M1A1 ≥ 2 terms, fit their p 's if $\Sigma p = 1$ NB: $\frac{19}{20} \times 3 = 2.85$ no mks
ia			With replacement:
ib			Original scheme $\frac{1}{20} + \frac{19}{20} \times \frac{1}{20} + (\frac{19}{20})^2 \times \frac{1}{20}$ or $1 - (\frac{19}{20})^2$ M1
ia			$(\frac{19}{20})^2$ or $(\frac{19}{20})^2 \times \frac{1}{20} + (\frac{19}{20})^2 \times \frac{19}{20}$ M1
b			Original scheme But NB ans 2.85(25...) M1A0M1A0
Total		13	



9i	$(1 - 0.12)^n$ $\frac{\log 0.05}{\log 0.88}$ $n = 24$	or $0.88^{23} = 0.052\dots$ or $0.88^{24} = 0.046\dots$	M1 M1 A1 3	Can be implied by 2 nd M1 allow $n - 1$ or $\log_{0.88} 0.05$ or 23.4(...) Ignore incorrect inequ or equals signs
ii	${}^6C_2 \times 0.88^4 \times 0.12^2$ $\times 0.12$ $= 0.0155$	(= 0.1295...)	M3 M1 A1 5	or $0.88^4 \times 0.12^2$ M2 or ${}^6C_2 \times 0.88^4 \times 0.12^2 + \text{extra}$ M2 or 2 successes in 6 trials implied or 6C_2 M1 dep \geq M1 $0.88^4 \times 0.12^2 \times 0.12$: M2M1 $0.88^4 \times 0.12^3$ M0M0A0 unless clear P(2 success in 6 trials) $\times 0.12$ in which case M2M1A0
Total			8	

Total 72 marks

4733 Probability & Statistics 2

1	$\frac{105.0 - \mu}{\sigma} = -0.7; \frac{110.0 - \mu}{\sigma} = -0.5$ <p>Solve: $\sigma = 25$ $\mu = 122.5$</p>	M1 A1 B1 M1 A1 A1	Standardise once, equate to Φ^{-1} , allow σ^2 Both correct including signs & σ , no cc (continuity correction), allow wrong z Both correct z -values. “1 –” errors: M1A0B1 Get either μ or σ by solving simultaneously σ a.r.t. 25.0 6 $\mu = 122.5 \pm 0.3$ or 123 if clearly correct, allow from σ^2 but <i>not</i> from $\sigma = -25$.
2	$Po(20) \approx N(20, 20)$ Normal approx. valid as $\lambda > 15$ $1 - \Phi\left(\frac{24.5 - 20}{\sqrt{20}}\right) = 1 - \Phi(1.006)$ $= 1 - 0.8427 = \mathbf{0.1573}$	M1 A1 B1 M1 A1 A1	Normal stated or implied (20, 20) or (20, $\sqrt{20}$) or (20, 20^2), can be implied “Valid as $\lambda > 15$ ”, or “valid as λ large” Standardise 25, allow wrong or no cc, $\sqrt{20}$ errors $1.0 < z \leq 1.01$ 6 Final answer, art 0.157
3	$H_0 : p = 0.6, H_1 : p < 0.6$ where p is proportion in population who believe it’s good value $R \sim B(12, 0.6)$ $\alpha: P(R \leq 4) = 0.0573 > 0.05$ $\beta: CR \text{ is } \leq 3 \text{ and } 4 > 3$ $p = 0.0153$ Do not reject H_0 . Insufficient evidence that the proportion who believe it’s good value for money is less than 0.6	B2 M1 A1 B1 B1 A1	Both, B2. Allow π % One error, B1, except x or \bar{x} or r or R : 0 B(12, 0.6) stated or implied, e.g. N(7.2, 2.88) <i>Not</i> $P(< 4)$ or $P(\geq 4)$ or $P(= 4)$ Must be using $P(\leq 4)$, or $P(> 4) < 0.95$ and binomial Must be using CR; explicit comparison needed 7 Correct conclusion, needs B(12,0.6) and ≤ 4 Contextualised, some indication of uncertainty [SR: N(7.2, ...) or Po(7.2): poss B2 M1A0] [SR: $P(< 4)$ or $P(= 4)$ or $P(\geq 4)$: B2 M1A0]
4 (i)	Eg “not all are residents”; “only those in street asked”	B1 B1	One valid relevant reason A definitely different valid relevant reason <i>Not</i> “not a random sample”, <i>not</i> “takes too long”
(ii)	Obtain list of whole population Number it sequentially Select using random numbers [Ignore method of making contact]	B1 B1 B1	“Everyone” or “all houses” must be implied <i>Not</i> “number it with random numbers” unless then “arrange in order of random numbers” SR: “Take a random sample”: B1 SR: Systematic: B1 B0, B1 if start randomly chosen
(iii)	Two of: α : Members of population equally likely to be chosen β : Chosen independently/randomly γ : Large sample (e.g. > 30)	B1 B1	One reason. NB : If “independent”, must be “chosen” independently, not “views are independent” Another reason. Allow “fixed sample size” but not both that and “large sample”. Allow “houses”

5	(i)	Bricks scattered at constant average rate & independently of one another	B1 B1	2	B1 for each of 2 different reasons, in context. (Treat “randomly” ≡ “singly” ≡ “independently”)
	(ii)	Po(12) $P(\leq 14) - P(\leq 7)$ [= .7720 - .0895] [or $P(8) + P(9) + \dots + P(14)$] = 0.6825	B1 M1 A1	3	Po(12) stated or implied Allow one out at either end or both, eg 0.617, or wrong column, but <i>not</i> from Po(3) nor, eg, .9105 - .7720 Answer in range [0.682, 0.683]
	(iii)	$e^{-\lambda} = 0.4$ $\lambda = -\ln(0.4)$ = 0.9163 Volume = $0.9163 \div 3 = \mathbf{0.305}$	B1 M1 A1 M1	4	This equation, aef, can be implied by, eg 0.9 Take ln, or 0.91 by T & I λ art 0.916 or 0.92, can be implied Divide their λ value by 3 [SR: Tables, eg $0.9 \div 3$: B1 M0 A0 M1]
6	(i)	33.6 $\frac{115782.84}{100} - 33.6^2$ [= 28.8684] $\times \frac{100}{99}$ = 29.16	B1 M1 M1 A1	4	33.6 clearly stated [not recoverable later] Correct formula used for biased estimate $\times \frac{100}{99}$, M's independent. Eg $\frac{\Sigma r^2}{99} [-33.6^2]$ SR B1 variance in range [29.1, 29.2]
	(ii)	$\bar{R} \sim N(33.6, 29.16/9)$ = $N(33.6, 1.8^2)$ $1 - \Phi\left(\frac{32 - 33.6}{\sqrt{3.24}}\right)$ [= $\Phi(0.8889)$] = 0.8130	M1 A1 M1 A1	4	Normal, their μ stated or implied Variance [their (i)] $\div 9$ [not $\div 100$] Standardise & use Φ , 9 used, answer > 0.5 , allow $\sqrt{\quad}$ errors, allow cc 0.05 but <i>not</i> 0.5 Answer, art 0.813
	(iii)	No, distribution of R is normal so that of \bar{R} is normal	B2	2	Must be saying this. Eg “9 is not large enough”: B0. Both: B1 max, unless saying that n is irrelevant.
7	(i)	$\frac{2}{9} \int_0^3 x^3(3-x) dx = \frac{2}{9} \left[\frac{3x^4}{4} - \frac{x^5}{5} \right]_0^3$ [= 2.7] - $(1\frac{1}{2})^2$ = $\frac{9}{20}$ or 0.45	M1 A1 B1 M1 A1	5	Integrate $x^2 f(x)$ from 0 to 3 [not for μ] Correct indefinite integral Mean is $1\frac{1}{2}$, soi [not recoverable later] Subtract their μ^2 Answer art 0.450
	(ii)	$\frac{2}{9} \int_0^{0.5} x(3-x) dx = \frac{2}{9} \left[\frac{3x^2}{2} - \frac{x^3}{3} \right]_0^{0.5}$ = $\frac{2}{27}$ AG	M1 A1	2	Integrate $f(x)$ between 0, 0.5, must be seen somewhere Correctly obtain given answer $\frac{2}{27}$, decimals other than 0.5 not allowed, 1 more line needed (eg [] = $\frac{1}{3}$)
	(iii)	$B(108, \frac{2}{27})$ $\approx N(8, 7.4074)$ $1 - \Phi\left(\frac{9.5 - 8}{\sqrt{7.4074}}\right)$ = $1 - \Phi(0.5511)$ = 0.291	B1 M1 A1 M1 A1 A1	6	$B(108, \frac{2}{27})$ seen or implied, eg Po(8) Normal, mean 8 variance (or SD) 200/27 or art 7.41 Standardise 10, allow $\sqrt{\quad}$ errors, wrong or no cc, needs to be using $B(108, \dots)$ Correct $\sqrt{\quad}$ and cc Final answer, art 0.291

(iv)	$\bar{X} \sim N(1.5, \frac{1}{240})$	B1 B1√ B1√ 3	Normal Mean their μ Variance or SD (their 0.45)/108 [not (8, 50/729)]	NB: not part (iii)
8 (i)	$H_0 : \mu = 78.0$ $H_1 : \mu \neq 78.0$ $z = \frac{76.4 - 78.0}{\sqrt{68.9/120}} = -2.1115$ > -2.576 or $0.0173 > 0.005$	B1 B1 M1 A1 B1	Both correct, B2. One error, B1, but x or \bar{x} : B0. Needs $\pm(76.4 - 78)/\sqrt{(\sigma^2/120)}$, allow $\sqrt{\quad}$ errors art -2.11 , or $p = 0.0173 \pm 0.0002$ Compare z with $(-)2.576$, or p with 0.005	
	$78 \pm z\sqrt{(68.9/120)}$ $= 76.048$ $76.4 > 76.048$	M1 A1√ B1	Needs 78 and 120, can be $-$ only Correct CV to 3 sf, $\sqrt{\quad}$ on z $z = 2.576$ and compare 76.4, allow from $78 \leftrightarrow 76.4$	
	Do not reject H_0 . Insufficient evidence that the mean time has changed	M1 A1√ 7	Correct comparison & conclusion, needs 120, "like with like", correct tail, \bar{x} and μ right way round Contextualised, some indication of uncertainty	
(ii)	$\frac{1}{\sqrt{68.9/n}} > 2.576$ $\sqrt{n} > 21.38,$ $n_{\min} = 458$ Variance is estimated	M1 M1 A1 B1 4	IGNORE INEQUALITIES THROUGHOUT Standardise 1 with n and 2.576, allow $\sqrt{\quad}$ errors, cc etc but <i>not</i> 2.326 Correct method to solve for \sqrt{n} (<i>not</i> from n) 458 only (<i>not</i> 457), or 373 from 2.326, signs correct Equivalent statement, allow "should use t ". In principle nothing superfluous, but "variance stays same" B1 bod	

Specimen Answers

Question 4: Part (i)

α	Takes too long/too slow	B0
β	Interviewing people in the street isn't a random sample	B0
γ	Many tourists so not representative	B1
δ	Those who don't shop won't have their views considered	B1
ε	Interviewers biased as to who they ask	B1
ζ	Views influenced by views of others	B1

Part (ii)

α	Choose a random sample of the town and ask their opinion	B1
β	Choose names at random from the town's phone book	B1
γ	A random number machine determines which house numbers should be used, and every street should have the same proportion of residents interviewed	B0B0B1
δ	Visit everyone door to door and give them a questionnaire	B1B0B0
ε	Assign everyone a number and select randomly	B1B0B0
ζ	Assign everyone a number and select using random numbers	B1B0B1
η	Ditto + "ignoring numbers that don't correspond to a resident"	B1B1B1
θ	Assign each eligible person a number and pick numbers from a hat	B1B1B0
ι	Put names of all residents into a hat and pick them out	B1B1B0

[NB: postal survey is biased]

Part (iii)

α	One person's view should not affect another's	B0
β	It is without bias	B0
γ	Results occur randomly	B0
δ	Should be asked if they are for or against (binomial testing)	B0
ε	It will survey a diverse group from different areas so should be representative	B0
ζ	Everyone's should be chose independently of everyone else	B1
η	The sample size must be large	B1
θ	Participants are chosen at random and independently from one another	B1 only

[though η & θ together would get B2]

Question 5 (i)

α	Number of bricks must always be the same	B0
β	Results occur randomly	B0
γ	The chance of a brick being in one place is always the same	B0
δ	Events must occur independently and at constant average rate	B0
ε	They must occur independently and at constant average rate	B1 only
ζ	Bricks' locations must be random and independent	<i>[effectively the same]</i> B1 only
η	Only one brick in any one place; bricks independent	<i>[effectively the same]</i> B1 only

4734 Probability & Statistics 3

Penalise 2 sf instead of 3 once only. Penalise final answer ≥ 6 sf once only.

1	(i)	$\int_0^1 \frac{2}{5}x^2 dx + \int_1^4 \frac{2}{5}\sqrt{x} dx$ $= \left[\frac{2x^3}{15} \right]_0^1 + \left[\frac{4x^{3/2}}{15} \right]_1^4 = 2$	M1 A1 A1	3	Attempt to integrate $xf(x)$, both parts added, limits Correct indefinite integrals Correct answer
	(ii)	$\int_2^4 \frac{2}{5\sqrt{x}} dx = \left[\frac{4\sqrt{x}}{5} \right]_2^4 = \frac{4}{5}(2-\sqrt{2}) \text{ or } 0.4686$	M1 A1 A1	3	Attempt correct integral, limits; needs “1 -” if $\mu < 1$ Correct indefinite integral, $\sqrt{\quad}$ on their μ Exact aef, or in range [0.468, 0.469]
2	(i)	<p>Po(0.5), Po(0.75) Po(0.7) and Po(0.9) $A + B \sim \text{Po}(1.6)$</p> <p>$P(A + B \geq 5) = 0.0237$ $B(20, 0.0237)$ $0.9763^{20} + 20 \times 0.9763^{19} \times 0.0237$ = 0.9195</p>	M1 A1 M1 A1 M1 A1√ A1	7	0.5, 0.75 scaled These Sum of Poissons used, can have wrong parameters 0.0237 from tables or calculator Binomial (20, their p), soi Correct expression, their p Answer in range [0.919, 0.92]
	(ii)	Bacteria should be independent in drugs; or sample should be random	B1	1	Any valid relevant comment, must be contextualised
3	(i)	<p>Sample mean = 6.486 $s^2 = 0.00073$</p> $6.486 \pm 2.776 \times \sqrt{\frac{0.00073}{5}}$ <p>(6.45, 6.52)</p>	B1 B1 M1 B1 A1A1	6	0.000584 if divided by 5 Calculate sample mean $\pm ts/\sqrt{5}$, allow 1.96, s^2 etc $t = 2.776$ seen Each answer, cwo (6.45246, 6.5195)
	(ii)	$2\pi \times \text{above} \quad [= (40.5, 41.0)]$	M1	1	
4	(i)	<p>$H_0: p_1 = p_2; H_1: p_1 \neq p_2$, where p_i is the proportion of all solvers of puzzle i Common proportion 39/80 $s^2 = 0.4875 \times 0.5125 / 20$ $(\pm) \frac{0.6 - 0.375}{0.1117} = (\pm) 2.013$</p> <p>$2.013 > 1.96$, or $0.022 < 0.025$ Reject H_0. Significant evidence that there is a difference in standard of difficulty</p>	B1 M1A1 B1 M1 A1√ M1 A1√	8	Both hypotheses correctly stated, allow eg \hat{p} [= 0.4875] [= 0.01249, $\sigma = 0.11176$] $(0.6 - 0.375)/s$ Allow 2.066√ from unpooled variance, $p = 0.0195$ Correct method and comparison with 1.96 or 0.025, allow unpooled, 1.645 from 1-tailed <i>only</i> Conclusion, contextualised, not too assertive
	(ii)	One-tail test used Smallest significance level 2.2(1)%	M1 A1	2	One-tailed test stated or implied by Φ (“2.013”), OK if off-scale; allow 0.022(1)

5 (i)	Numbers of men and women should have normal dists; with equal variance; distributions should be independent	B1 B1 B1 3	Context & 3 points: 2 of these, B1; 3, B2; 4, B3. [Summary data: 14.73 49.06 52.57 16.24 62.18 66.07]
(ii)	$H_0: \mu_M = \mu_W; \quad H_1: \mu_M \neq \mu_W$ $3992 - \frac{221^2}{15} + 5538 - \frac{276^2}{17} \quad [\approx 1793]$ $1793 / (14 + 16) = 59.766$ $(\pm) \frac{221/15 - 276/17}{\sqrt{59.766(\frac{1}{15} + \frac{1}{17})}} = (-)0.548$ <p>Critical region: $t \geq 2.042$ Do not reject H_0. Insufficient evidence of a difference in mean number of days</p>	B1 M1 A1 A1 M1 A1√ A1 B1 M1 A1√ 10	Both hypotheses correctly stated Attempt at this expression (see above) Either 1793 or 30 Variance estimate in range [59.7, 59.8] (or $\sqrt{= 7.73}$) Standardise, allow wrong (but not missing) $1/n$ Correct formula, allow $s^2(\frac{1}{15} + \frac{1}{17})$ or $(\frac{1}{15} + \frac{1}{17})$, allow 14 & 16 in place of 15, 17; 0.548 or -0.548 2.042 seen Correct method and comparison type, must be t , allow 1-tail; conclusion, in context, not too assertive
(iii)	Eg Samples not indep't so test invalid	B1 1	Any relevant valid comment, eg "not representative"

6 (i)	$F(0) = 0, F(\pi/2) = 1$ Increasing	B1 B1 2	Consider both end-points Consider F between end-points, can be asserted
(ii)	$\sin^4(Q_1) = 1/4$ $\sin(Q_1) = 1/\sqrt{2}$ $Q_1 = \pi/4$	M1 A1 A1 3	Can be implied. Allow decimal approximations Or 0.785(4)
(iii)	$G(y) = P(Y \leq y) = P(T \leq \sin^{-1} y)$ $= F(\sin^{-1} y)$ $= y^4$ $g(y) = \begin{cases} 4y^3 & 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$	M1 A1 A1 M1 A1 5	Ignore other ranges Differentiate G(y) Function and range stated, allow if range given in G
(iv)	$\int_0^1 \frac{4}{1+2y} dy = [2 \ln(1+2y)]_0^1$ $= 2 \ln 3$	M1 A1 A1 3	Attempt $\int \frac{g(y)}{y^3+2y^4} dy; \int_0^1 \frac{4}{1+2y} dy$ Or 2.2, 2.197 or better
7 (i) α	$\Phi\left(\frac{8.084-8.592}{0.7534}\right) = \Phi(-0.674) = 0.25$ $\Phi(0) - \Phi(\text{above}) = 0.25$ $P(8.592 \leq X \leq 9.1) = \text{same by symmetry}$	M1 A1 A1 A1 4	Standardise once, allow $\sqrt{\quad}$ confusions, ignore sign Obtain 0.25 for one interval For a second interval, justified, eg using $\Phi(0) = 0.5$ For a third, justified, eg "by symmetry"
or β	$\frac{x-8.592}{0.7534} = 0.674$ $x = 8.592 \pm 0.674 \times 0.7534$ $= (8.084, 9.100)$	M1A1 A1A1	[from probabilities to ranges] A1 for art 0.674
(ii)	H_0 : normal distribution fits data All E values $50/4 = 12.5$ $\chi^2 = \frac{4.5^2 + 9.5^2 + 1.5^2 + 3.5^2}{12.5} = 10$ $10 > 7.8794$ Reject H_0 . Significant evidence that normal distribution is not a good fit.	B1 B1 M1 A1 B1 M1 A1 $\sqrt{7}$	Not N(8.592, 0.7534). Allow "it's normally distributed" [Yates: 8.56: A0] CV 7.8794 seen Correct method, incl. formula for χ^2 and comparison, allow wrong ν Conclusion, in context, not too assertive
(iv)	$8.592 \pm 2.576 \times \frac{0.7534}{\sqrt{49}}$ $(8.315, 8.869)$	M1 A1 A1 3	Allow $\sqrt{\quad}$ errors, wrong σ or z , allow 50 Correct, including $z = 2.576$ or $t_{49} = 2.680$, not 50 In range [8.31, 8.32] and in range (8.86, 8.87], even from 50, or (8.306, 8.878) from t_{49}

4735 Probability & Statistics 4


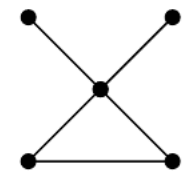
<p>1</p>	$M_{X_1+X_2}(t) = (e^{\mu_1 t + \frac{1}{2}\sigma_1^2 t^2})(e^{\mu_2 t + \frac{1}{2}\sigma_2^2 t^2})$ $= e^{(\mu_1 + \mu_2)t + \frac{1}{2}(\sigma_1^2 + \sigma_2^2)t^2} \quad oe$ <p>$X_1 + X_2 \sim$ Normal distribution with mean $\mu_1 + \mu_2$, variance $\sigma_1^2 + \sigma_2^2$</p>	<p>M1 A1 A1 A1A1 5 {5}</p>	<p>MGF of sum of independent RVs</p> <p>No suffices:- Allow M1A0A1A0A0</p>
<p>2 (i)</p>	<p>Non-parametric test used when the distribution of the variable in question is unknown</p>	<p>B1 1</p>	
<p>(ii)</p>	<p>$H_0: m_{V-A} = 0, H_1: m_{V-A} \neq 0$ where m_{V-A} is the median of the population differences</p> <p>Difference and rank, bottom up $P = 65 \quad Q = 13$ $T = 13$ Critical region: $T \leq 13$ 13 is inside the CR so reject H_0 and accept that there is sufficient evidence at the 5% significance level that the medians differ</p> <p>Use B(12, 0.5) $P(\leq 4) = 0.1938$ or CR = {0,1,2,10,11,12} > 0.025, accept that there is insufficient evidence, etc CWO</p>	<p>B1 M1 A1 B1 M1 A1 M1 A1 A1 9</p>	<p>Allow $m_V = m_A$ etc</p> <p>Allow $P > Q$ stated</p> <p>Penalise over-assertive conclusions once only.</p> <p>Or 4 not in CR</p>
<p>(iii)</p>	<p>Wilcoxon test is more powerful than the sign test</p>	<p>B1 1 {11}</p>	<p>Use more information, more likely to reject NH</p>
<p>3(i)</p>	<p>$A + B$ $= \int_{-\infty}^0 e^{2x} e^{xt} dx + \int_0^{\infty} e^{-2x} e^{xt} dx$ $= \left[\frac{1}{2+t} e^{(2+t)x} \right]_{-\infty}^0 + \left[-\frac{1}{2-t} e^{-(2-t)x} \right]_0^{\infty}$ $= 1/(2+t) + 1/(2-t)$ $= 4/(4-t^2)$ AG $t < -2, A$ infinite; $t > 2, B$ infinite</p>	<p>M1 B1 B1 A1 B1 5</p>	<p>Added, correct limits</p> <p>Correct integrals</p> <p>Allow sensible comments about denom of $M(t)$</p>
<p>(ii)</p>	<p>Either: $4/(4-t^2) = (1 - 1/4t^2)^{-1}$ $= 1 + 1/4t^2 + \dots$</p>	<p>M1 A1</p>	<p>Expand</p>
	<p>Or: $M'(t) = 8t/(4-t^2)^2$ $M''(t) = 8/(4-t^2)^2 + t \times \dots$</p>	<p>M1 A1</p>	<p>M1 A1</p>
	<p>$E(X) = 0$ $\text{Var}(X) = 2 \times 1/4 - 0 = 1/2$</p>	<p>M1 A1 4 {9}</p>	<p>For $M''(0) - [M'(0)]^2$ or equivalent $0.5 - 0 = 0.5$</p>

4	$G(1)=1$ $[a+b=1]$ (i) $G'(1)=-0.7$ $[-a+2b=-0.7]$ Solve to obtain $a=0.9, b=0.1$	M1 M1 M1 A1	4	
(ii)	$G''(t) [=1.8t^2 + 0.2]$ and $G''(1) + G'(1) - [G'(1)^2]$ used $\text{Var} = 2 - 0.7 - 0.7^2 = 0.81$	M1 A1	2	
(iii)	$[(0.9 + 0.1t^3)/t]^{10}$ Method to obtain coefficient of t^{-7} $10 \times 0.9^9 \times 0.1$ $= 0.387$ to 3SF	M1 M1 A1 ft A1	4 {10}	$[(a + bt^3)/t]^{10}$ For both Use of MGF. $10a^9b$
5	Marginal dist of X_A : 0.30 0.45 0.15 0.10 (i) $E = 0.45 + 0.3 + 0.3 = 1.05$ $\text{Var} = 0.45 + 0.6 + 0.9 - 1.05^2$ $= 0.8475$	B1 B1 B1	3	
(ii)	Consider a particular case to show $P(X_A \text{ and } X_B) \neq P(X_A)P(X_B)$ So X_A and X_B are not independent	M1 A1	2	Or $E(X_A), E(X_B)$ and $E(X_A X_B)$ 1.05, 1.15, 1.09; $E(X_A)E(X_B) = 1.0275$, ft on wrong $E(X_A)$
(iii)	$\text{Cov} = E(X_A X_B) - E(X_A)E(X_B)$ $= 1.09 - 1.15 \times 1.05 = -0.1175$ $\text{Var}(X_A - X_B) = \text{Var}(X_A) + \text{Var}(X_B) - 2\text{Cov}(X_A, X_B)$ $= 1.91$	M1 A1ft M1 A1	4	Or from distribution of $X_A - X_B$ Wrong $E(X_A)$
(iv)	Requires $P(X_A, X_B)/P(X_A + X_B = 1)$ $= 0.13/(0.16 + 0.13)$ $= 13/29$ $= 0.448$	M1 A1A1 A1	4 {13}	

6 (i)	$\int_a^\infty x e^{-(x-a)} dx = \left[-x e^{-(x-a)} \right]_a^\infty + \int_a^\infty e^{-(x-a)} dx$ $= a + [- e^{-(x-a)}]$ $= a + 1 \quad \text{AG}$	M1B1 A1	3	Correct limits needed for M1; no, or incorrect, limits allowed for B1
(ii)	$E(T_1) = (a+1) + 2(a+1) - 2(a+1) - 1$ $= a$ $E(T_2) = \frac{1}{4}(a+1+a+1) + (n-2)(a+1)/[2(n-2)] - 1$ $= a$ <p>(So both are unbiased estimators of a)</p>	M1 A1 M1 A1	4	
(iii)	$\sigma^2 = \text{Var}(X)$ $\text{Var}(T_1) = (1 + 4 + 1 + 1)\sigma^2 = 7\sigma^2$ $\text{Var}(T_2) = 2\sigma^2/16 + (n-2)\sigma^2/[2(n-2)^2]$ $= n\sigma^2/[8(n-2)] \text{ oe}$ <p>This is clearly $< 7\sigma^2$, so T_2 is more efficient</p>	M1 A1 B1 A1	4	
(iv)	eg $\frac{1}{n}(X_1 + X_2 + \dots + X_n) - 1$	B2	2	B1 for sample mean
			{13}	
7 (i)	<p>D denotes "The person has the disease"</p> <p>(a) $P(D) = p, P(D') = 1 - p,$ $P(+ D) = 0.98, P(+ D') = 0.08$ $P(+) = p \times 0.98 + 0.08 \times (1 - p)$ $= 0.08 + 0.9p$ $P(D +) = P(+ D)P(D)/P(+)$ $= 0.98p/(0.08 + 0.9p)$</p> <p>(b) $P(D') \times P(+ D') + P(D) \times P(- D)$ $= 0.08 - 0.06p$</p>	M1 M1 A1 M1 A1	5	Use conditional probability
(ii)	$P(++) = 0.98^2 \times p + 0.08^2 \times (1 - p)$ $P(D ++) = 0.9604p/(0.954p + 0.0064)$	M1 A1	2	
(iii)	<p>Expected number with 2 tests: $24000 \times 0.0809 = a$</p> <p>Expected number with 1 test: $24000 \times 0.9191 = b$</p> <p>Expected total cost = $\pounds(10a + 5b)$ $= \pounds 129\,708$</p>	M1 M1 M1 A1	4	Or: $0.08 + 0.9 \times 0.001$ oe $\times 5 \times 24000$ $+ 5 \times 24000$ (dep 1 st M1) Or $\pounds 130\,000$
			{11}	

4736 Decision Mathematics 1

1 (i)	[43 172 536 17 314 462 220 231] 43 172 536 17 220 314 462 231	M1 M1 A1	First folder correct Second folder correct All correct (cao)	[3]
(ii)	536 462 314 231 220 172 43 17 536 462 314 231 220 172 43 17	B1 M1 A1	List sorted into decreasing order seen (cao) [Follow through from a decreasing list with no more than 1 error or omission] First folder correct All correct	[3]
(iii)	$(5000 \div 500)^2 \times 1.3$ = 130 seconds	M1 A1	$10^2 \times 1.3$ or any equivalent calculation Correct answer, with units	[2]
Total = 8				

2 (i)	The sum of the orders must be even, (but $1+2+3+3 = 9$ which is odd).	B1	There must be an even number of odd nodes.	[1]
(ii) a	eg 	M1 A1	A graph with five vertices that is neither connected nor simple Vertex orders 1, 1, 2, 2, 4	[2]
b	Because it is not connected	B1	You cannot get from one part of the graph to the other part.	[1]
c	eg 	B1	A connected graph with vertex orders 1, 1, 2, 2, 4 (Need not be simple)	[1]
(iii) a	There are five arcs joined to A . Either Ann has met (at least) three of the others or she has met two or fewer, in which case there are at least three that she has not met. In the first case at least three of the arcs joined to A are blue, in the second case at least three of the arcs joined to A are red.	M1 A1	A reasonable attempt (for example, identifying that there are five arcs joined to A) A convincing explanation (this could be a list of the possibilities or a well reasoned explanation)	[2]
b	If any two of Bob, Caz and Del have met one another then B , C and D form a blue triangle with A . Otherwise B , C and D form a red triangle.	M1 A1	A reasonable or partial attempt (using A with B , C , D) A convincing explanation (explaining both cases fully)	[2]
Total = 9				

3 (i)	$y \geq x$ $x + y \leq 8$ $x \geq 1$	M1 M1 M1 A1	Line $y = x$ in any form Line $x + y = 8$ in any form Line $x = 1$ in any form All inequalities correct [Ignore extra inequalities that do not affect the feasible region]	[4]
(ii)	(1, 1), (1, 7), (4, 4)	M1 A1	Any two correct coordinates All three correct [Extra coordinates given \Rightarrow M1, A0]	[2]
(iii)	$(1, 7) \square 23$ $(4, 4) \square 20$ At optimum, $x = 1$ and $y = 7$ Maximum value = 23	M1 A1 A1	Follow through if possible Testing vertices or using a line of constant profit (may be implied) Accept (1, 7) identified 23 identified	[3]
(iv)	$2 \times 1 + k \times 7 \geq 2 \times 4 + k \times 4$ $\square k \geq 2$	M1 A1	$2 + 7k$ or implied, or using line of gradient $-\frac{2}{k}$ Greater than or equal to 2 (cao) [$k > 2 \Rightarrow$ M1, A0]	[2]
Total =				11

<p>4 (i)</p> <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; padding: 2px;">1 0</div> <div style="border: 1px solid black; padding: 2px;">6 8</div> </div> <p style="text-align: center;"><i>A</i> <i>E</i></p> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="border: 1px solid black; padding: 2px;">2 2</div> <div style="border: 1px solid black; padding: 2px;">4 5</div> <div style="border: 1px solid black; padding: 2px;">5 6.5</div> <div style="border: 1px solid black; padding: 2px;">7 9.5</div> </div> <p style="text-align: center;"><i>B</i> <i>D</i> <i>F</i> <i>H</i></p> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="border: 1px solid black; padding: 2px;">3 4.5</div> <div style="border: 1px solid black; padding: 2px;">14 13.5 10.5</div> </div> <p style="text-align: center;"><i>C</i> <i>G</i></p> <p style="margin-top: 10px;">Route = $A-B-D-F-H$ Length = 9.5 miles</p>	<p>M1 M1 A1</p> <p>B1</p> <p>B1</p> <p>B1 B1</p>	<p>Both 6 and 5 shown at <i>D</i> [5 may appear as perm label only]</p> <p>14, 13.5 and 10.5 shown at <i>G</i></p> <p>No extra temporary labels</p> <p>All temporary labels correct [condone perm values only appearing as perm labels] [Dep on both M marks]</p> <p>All permanent labels correct [may omit <i>G</i>, but if given it must be correct]</p> <p>Order of labelling correct [may omit <i>G</i> but if given it must be correct]</p> <p>cao cao</p>	<p>[7]</p>
<p>(ii)</p>	<p>Route Inspection problem</p>	<p>B1</p>	<p>Accept Chinese Postman [1]</p>
<p>(iii)</p>	<p>Odd nodes: <i>A, D, E</i> and <i>H</i> $AD = 5$ $AE = 8$ $AH = 9.5$ $EH = \frac{5}{10}$ $DH = \frac{4.5}{12.5}$ $DE = \frac{3.5}{13.0}$</p> <p>Repeat AD ($A-B-D$) and EH ($E-F-H$) Length = $67.5 + 10$ = 77.5 miles</p>	<p>B1 M1 A1</p> <p>M1</p> <p>A1</p>	<p>Identifying or using A, D, E, H</p> <p>Attempting at least one pairing</p> <p>At least one correct pairing or correct total</p> <p>Adding their 10 to 67.5</p> <p>77.5 (cao)</p> <p>[5]</p>
<p>(iv)</p>	<p>Repeat arcs EF and FD $3.5 + 67.5 = 71$ miles</p>	<p>B1 B1</p>	<p>cao [NOT DE or $D-F-E$] cao</p> <p>[2]</p>
<p>(v)</p>	<p>$A-B-C-G-F-D$ then method stalls <i>E</i> and <i>H</i> are missed out</p>	<p>B1</p>	<p>Showing route as far as <i>D</i> and then explaining the problem</p> <p>[1]</p>
<p>(vi)</p>	<p>$C-B-A-D-F-E-H-G-C$ 37.5 miles</p>	<p>M1 A1 B1</p>	<p>[If final <i>C</i> is missing \Rightarrow M1, A0] [A diagram needs arrows for A1] 37.5 (cao)</p> <p>[3]</p>
<p>(vii)</p>	<div style="text-align: center;"> </div> <p>Nodes: $B C D F E H G$ Weight = 16 miles</p> <p>[Two shortest arcs from <i>A</i> are AB and AD] $2 + 6 + 16$ Lower bound = 24 miles</p>	<p>M1 A1</p> <p>B1 B1</p> <p>M1 A1</p>	<p>A spanning tree on reduced network (may show AB, AD)</p> <p>Correct minimum spanning tree marked, with no extra arcs</p> <p>cao cao</p> <p>8 + their 16 (or implied) cao</p> <p>[6]</p>
<p>Total = 25</p>			

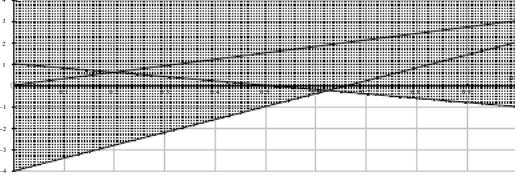
5 (i)	$15x+15y+30z \leq 9000$ [divide through by 15 to get $x+y+2z \leq 600$ as given] Stamping out: $5x+8y+10z \leq 3600$ Fixing pin: $50x+50y+50z \leq 25000$ $\square x + y + z \leq 500$ Checking: $100x+50y+20z \leq 10000$ $\square 10x+5y+2z \leq 1000$	B1	$15x+15y+30z \leq 9000$																																																							
		B1	$5x+8y+10z \leq 3600$																																																							
		B1	$x + y + z \leq 500$																																																							
		B1	$10x+5y+2z \leq 1000$																																																							
(ii)	x, y and z are non-negative	B1	$x \geq 0, y \geq 0$ and $z \geq 0$	[1]																																																						
(iii)	$(P =) 4x + 3y + z$	B1	cao	[1]																																																						
(iv)	<table border="1"> <thead> <tr> <th>P</th> <th>x</th> <th>y</th> <th>z</th> <th>s</th> <th>t</th> <th>u</th> <th>v</th> <th>RHS</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>-4</td> <td>-3</td> <td>-1</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <td>0</td> <td>1</td> <td>1</td> <td>2</td> <td>1</td> <td>0</td> <td>0</td> <td>0</td> <td>600</td> </tr> <tr> <td>0</td> <td>5</td> <td>8</td> <td>10</td> <td>0</td> <td>1</td> <td>0</td> <td>0</td> <td>3600</td> </tr> <tr> <td>0</td> <td>1</td> <td>1</td> <td>1</td> <td>0</td> <td>0</td> <td>1</td> <td>0</td> <td>500</td> </tr> <tr> <td>0</td> <td>10</td> <td>5</td> <td>2</td> <td>0</td> <td>0</td> <td>0</td> <td>1</td> <td>1000</td> </tr> </tbody> </table>	P	x	y	z	s	t	u	v	RHS	1	-4	-3	-1	0	0	0	0	0	0	1	1	2	1	0	0	0	600	0	5	8	10	0	1	0	0	3600	0	1	1	1	0	0	1	0	500	0	10	5	2	0	0	0	1	1000	B1 B1 M1 A1	Follow through if reasonable -4 -3 -1 in objective row Correct use of slack variables 1 1 2 and 600 correct All constraint rows correct Accept variations in order of rows and columns	[4]
P	x	y	z	s	t	u	v	RHS																																																		
1	-4	-3	-1	0	0	0	0	0																																																		
0	1	1	2	1	0	0	0	600																																																		
0	5	8	10	0	1	0	0	3600																																																		
0	1	1	1	0	0	1	0	500																																																		
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(v)	Pivot on the 10 in the x -column	B1	Correct choice of pivot from x -column																																																							
	<table border="1"> <tbody> <tr> <td>1</td> <td>0</td> <td>-1</td> <td>-0.2</td> <td>0</td> <td>0</td> <td>0</td> <td>0.4</td> <td>400</td> </tr> <tr> <td>0</td> <td>0</td> <td>0.5</td> <td>1.8</td> <td>1</td> <td>0</td> <td>0</td> <td>-0.1</td> <td>500</td> </tr> <tr> <td>0</td> <td>0</td> <td>5.5</td> <td>9</td> <td>0</td> <td>1</td> <td>0</td> <td>-0.5</td> <td>3100</td> </tr> <tr> <td>0</td> <td>0</td> <td>0.5</td> <td>0.8</td> <td>0</td> <td>0</td> <td>1</td> <td>-0.1</td> <td>400</td> </tr> <tr> <td>0</td> <td>1</td> <td>0.5</td> <td>0.2</td> <td>0</td> <td>0</td> <td>0</td> <td>0.1</td> <td>100</td> </tr> </tbody> </table>	1	0		-1	-0.2	0	0	0	0.4	400	0	0	0.5	1.8	1	0	0	-0.1	500	0	0	5.5	9	0	1	0	-0.5	3100	0	0	0.5	0.8	0	0	1	-0.1	400	0	1	0.5	0.2	0	0	0	0.1	100	M1 A1	[Follow through their tableau and valid pivot if possible: no negative values in RHS column and P value has not decreased] Pivot row correct Other rows correct	[3]								
	1	0	-1		-0.2	0	0	0	0.4	400																																																
	0	0	0.5		1.8	1	0	0	-0.1	500																																																
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	0	1	0.5		0.2	0	0	0	0.1	100																																																
	Pivot on 0.5 in the last row of y -column	B1	Correct choice of pivot from y -column																																																							
	<table border="1"> <tbody> <tr> <td>1</td> <td>2</td> <td>0</td> <td>0.2</td> <td>0</td> <td>0</td> <td>0</td> <td>0.6</td> <td>600</td> </tr> <tr> <td>0</td> <td>-1</td> <td>0</td> <td>1.6</td> <td>1</td> <td>0</td> <td>0</td> <td>-0.2</td> <td>400</td> </tr> <tr> <td>0</td> <td>-11</td> <td>0</td> <td>6.8</td> <td>0</td> <td>1</td> <td>0</td> <td>-1.6</td> <td>2000</td> </tr> <tr> <td>0</td> <td>-1</td> <td>0</td> <td>0.6</td> <td>0</td> <td>0</td> <td>1</td> <td>-0.2</td> <td>300</td> </tr> <tr> <td>0</td> <td>2</td> <td>1</td> <td>0.4</td> <td>0</td> <td>0</td> <td>0</td> <td>0.2</td> <td>200</td> </tr> </tbody> </table>	1	2		0	0.2	0	0	0	0.6	600	0	-1	0	1.6	1	0	0	-0.2	400	0	-11	0	6.8	0	1	0	-1.6	2000	0	-1	0	0.6	0	0	1	-0.2	300	0	2	1	0.4	0	0	0	0.2	200	M1 A1	[Follow through their tableau and valid pivot if possible] Pivot row correct Other rows correct	[3]								
	1	2	0		0.2	0	0	0	0.6	600																																																
0	-1	0	1.6	1	0	0	-0.2	400																																																		
0	-11	0	6.8	0	1	0	-1.6	2000																																																		
0	-1	0	0.6	0	0	1	-0.2	300																																																		
0	2	1	0.4	0	0	0	0.2	200																																																		
$x = 0, y = 200, z = 0, P = 600$ Make 20 000 metallic badges (and no laminated badges or plastic badges)	B1	Interpretation of their x, y and z values in context (may imply zero entries)																																																								
To give a profit of £600	B1	Interpretation of their P value in context																																																								
6000 seconds (100 min) of printing time not used, 2000 seconds (33 min 20 sec) of stamping out time not used, 15000 seconds (250 min) of fixing pin time not used. All the checking time is used	B1	Interpretation of their slack variable values	[3]																																																							
Total = 19																																																										

4737 Decision Mathematics 2

<p>1(a) (i)</p>		<p>B1</p>	<p>A correct bipartite graph</p>	<p>[1]</p>
<p>(ii)</p>		<p>B1</p>	<p>A second bipartite graph showing the incomplete matching correctly</p>	<p>[1]</p>
<p>(iii)</p>	<p>$E = F - A = H - D = K$</p> <p>Fiona = Egg and tomato $F = E$ Gwen = Beef and horseradish $G = B$ Helen = Avocado and bacon $H = A$ Jack = Chicken and stuffing $J = C$ Mr King = Duck and plum sauce $K = D$</p>	<p>B1 B1</p>	<p>This path in any reasonable form</p> <p>This complete matching</p>	<p>[2]</p>
<p>(iv)</p>	<p>Interchange Gwen and Jack $F = E \quad G = C \quad H = A \quad J = B \quad K = D$</p>	<p>B1</p>	<p>This complete matching</p>	<p>[1]</p>

(b)	Reduce rows	<table border="1"> <thead> <tr> <th></th> <th>F</th> <th>G</th> <th>H</th> <th>J</th> <th>K</th> </tr> </thead> <tbody> <tr> <td>L</td> <td>7</td> <td>7</td> <td>7</td> <td>7</td> <td>0</td> </tr> <tr> <td>M</td> <td>2</td> <td>6</td> <td>4</td> <td>2</td> <td>0</td> </tr> <tr> <td>N</td> <td>8</td> <td>8</td> <td>8</td> <td>6</td> <td>0</td> </tr> <tr> <td>O</td> <td>1</td> <td>3</td> <td>2</td> <td>1</td> <td>0</td> </tr> <tr> <td>P</td> <td>6</td> <td>9</td> <td>7</td> <td>5</td> <td>0</td> </tr> </tbody> </table>		F	G	H	J	K	L	7	7	7	7	0	M	2	6	4	2	0	N	8	8	8	6	0	O	1	3	2	1	0	P	6	9	7	5	0	M1 Substantially correct attempt to reduce rows	[3]	
		F	G	H	J	K																																			
	L	7	7	7	7	0																																			
	M	2	6	4	2	0																																			
	N	8	8	8	6	0																																			
	O	1	3	2	1	0																																			
	P	6	9	7	5	0																																			
	Reduce columns	<table border="1"> <thead> <tr> <th></th> <th>F</th> <th>G</th> <th>H</th> <th>J</th> <th>K</th> </tr> </thead> <tbody> <tr> <td>L</td> <td>6</td> <td>4</td> <td>5</td> <td>6</td> <td>0</td> </tr> <tr> <td>M</td> <td>1</td> <td>3</td> <td>2</td> <td>1</td> <td>0</td> </tr> <tr> <td>N</td> <td>7</td> <td>5</td> <td>6</td> <td>5</td> <td>0</td> </tr> <tr> <td>O</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <td>P</td> <td>5</td> <td>6</td> <td>5</td> <td>4</td> <td>0</td> </tr> </tbody> </table>		F	G	H	J	K	L	6	4	5	6	0	M	1	3	2	1	0	N	7	5	6	5	0	O	0	0	0	0	0	P	5	6	5	4	0			M1 Substantially correct attempt to reduce columns
		F	G	H	J	K																																			
	L	6	4	5	6	0																																			
M	1	3	2	1	0																																				
N	7	5	6	5	0																																				
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	F	G	H	J	K																																				
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O	0	0	0	0	0																																				
P	5	6	5	4	0																																				
Augment by 1	<table border="1"> <thead> <tr> <th></th> <th>F</th> <th>G</th> <th>H</th> <th>J</th> <th>K</th> </tr> </thead> <tbody> <tr> <td>L</td> <td>5</td> <td>3</td> <td>4</td> <td>5</td> <td>0</td> </tr> <tr> <td>M</td> <td>0</td> <td>2</td> <td>1</td> <td>0</td> <td>0</td> </tr> <tr> <td>N</td> <td>6</td> <td>4</td> <td>5</td> <td>4</td> <td>0</td> </tr> <tr> <td>O</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <td>P</td> <td>4</td> <td>5</td> <td>4</td> <td>3</td> <td>0</td> </tr> </tbody> </table>		F	G	H	J	K	L	5	3	4	5	0	M	0	2	1	0	0	N	6	4	5	4	0	O	0	0	0	0	1	P	4	5	4	3	0	M1 Substantially correct attempt at augmenting			
	F	G	H	J	K																																				
L	5	3	4	5	0																																				
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M	0	2	1	0	0																																				
N	6	4	5	4	0																																				
O	0	0	0	0	1																																				
P	4	5	4	3	0																																				
Augment by 3	<table border="1"> <thead> <tr> <th></th> <th>F</th> <th>G</th> <th>H</th> <th>J</th> <th>K</th> </tr> </thead> <tbody> <tr> <td>L</td> <td>2</td> <td style="background-color: black; color: white;">0</td> <td>1</td> <td>2</td> <td>0</td> </tr> <tr> <td>M</td> <td style="background-color: black; color: white;">0</td> <td>2</td> <td>1</td> <td>0</td> <td>3</td> </tr> <tr> <td>N</td> <td>3</td> <td>1</td> <td>2</td> <td>1</td> <td style="background-color: black; color: white;">0</td> </tr> <tr> <td>O</td> <td>0</td> <td>0</td> <td style="background-color: black; color: white;">0</td> <td>0</td> <td>4</td> </tr> <tr> <td>P</td> <td>1</td> <td>2</td> <td>1</td> <td style="background-color: black; color: white;">0</td> <td>0</td> </tr> </tbody> </table>		F	G	H	J	K	L	2	0	1	2	0	M	0	2	1	0	3	N	3	1	2	1	0	O	0	0	0	0	4	P	1	2	1	0	0	M1 Substantially correct attempt at augmenting (by more than 1 in a single step)			
	F	G	H	J	K																																				
L	2	0	1	2	0																																				
M	0	2	1	0	3																																				
N	3	1	2	1	0																																				
O	0	0	0	0	4																																				
P	1	2	1	0	0																																				
		A1 Augmenting correctly																																							
Lemon = Gwen Mandarin = Fiona Nectarine = Mr King Orange = Helen Peach = Jack		B1 Correct allocation																																							
			[3]																																						
Total = 13																																									

2 (i)	Stage	State	Action	Working	Suboptimal maxima	B1	Structure of table correct		
	2	0	0	7	7	M1	Stage and state values correct	[3]	
		1	0	6	6				
		2	0	8	8				
	1	0	0	5 + 7 = 12	12	A1	Action values correct	[3]	
			1	6 + 6 = 12					
		1	1	0	4 + 7 = 11	14	B1		Working backwards from stage 2 7, 6, 8 correct in suboptimal maxima column for stage 2
				1	5 + 6 = 11				
				2	6 + 8 = 14				
			0	10 + 7 = 17	17				
2	1	9 + 6 = 15							
		2	6 + 8 = 14						
0	0	0	8 + 12 = 20	24	A1	Sums correct for stage 1	[3]		
		1	9 + 14 = 23						
		2	7 + 17 = 24						
Maximum route = (0;0) - (1;2) - (2;0) - (3;0) Weight = 24						B1	Correct route from (0; 0) to (3; 0)	[3]	
						B1	24 cao	[2]	
(ii)	<p>Minimum completion time = 24 Critical activities: C, I, L</p>					B1	Assigning A to N appropriately		
						M1	Substantially correct forward pass		
						A1	Forward pass correct		
						M1	Substantially correct backward pass		
						A1	Backward pass correct		
						A1	24 (cao)		
						B1	C, I, L (cao)		
						B1		[7]	
(iii)	The critical path is the maximum path The critical activities form a continuous path with no slack, ie the longest path					M1	Same path is found in both		
						A1	Recognition of why the solutions are the same, in general	[2]	
Total = 20									

3 (i)	For each pairing, the total of the points is 10. Subtracting 5 from each makes the total 0. Eg 3 points and 7 points \Rightarrow scores of -2 and +2	M1 A1	Sum of points is 10 So sum of scores is zero A specific example earns M1 only	[2]																																		
(ii)	W scores -1 P has 6 points and W has 4 points	B1 B1	-1 6 and 4	[2]																																		
(iii)	W is dominated by Y $-1 < 1$, $-3 < -2$ and $1 < 2$	B1 B1	Y These three comparisons in any form	[2]																																		
(iv)	<table style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th colspan="2"></th> <th colspan="3">Collies</th> <th></th> </tr> <tr> <th colspan="2"></th> <th>X</th> <th>Y</th> <th>Z</th> <th>row min</th> </tr> </thead> <tbody> <tr> <th rowspan="3">Rovers</th> <th>P</th> <td>2</td> <td>-1</td> <td>3</td> <td>-1</td> </tr> <tr> <th>Q</th> <td>1</td> <td>-3</td> <td>-1</td> <td>-3</td> </tr> <tr> <th>R</th> <td>-4</td> <td>1</td> <td>0</td> <td>-4</td> </tr> <tr> <th colspan="2">col max</th> <td>2</td> <td>1</td> <td>3</td> <td></td> </tr> </tbody> </table> <p>Play-safe for Rovers is P Play-safes for Collies is Y</p>			Collies						X	Y	Z	row min	Rovers	P	2	-1	3	-1	Q	1	-3	-1	-3	R	-4	1	0	-4	col max		2	1	3		M1 A1 A1	Determining row minima and column maxima, or equivalent P Y	[3]
		Collies																																				
		X	Y	Z	row min																																	
Rovers	P	2	-1	3	-1																																	
	Q	1	-3	-1	-3																																	
	R	-4	1	0	-4																																	
col max		2	1	3																																		
(v)	$2p - 4(1-p) = 6p - 4$ Y gives $1 - 2p$ Z gives $3p$	B1 B1	$6p - 4$ in simplified form Both $1 - 2p$ and $3p$ in any form	[2]																																		
(vi)	 <p>$6p - 4 = 1 - 2p \Rightarrow p = \frac{5}{8}$</p>	B1 M1 A1	Their lines drawn correctly on a reasonable scale Solving the correct pair of equations or using graph correctly $\frac{5}{8}$, 0.625, cao	[3]																																		
(vii)	Add 4 throughout matrix to make all values non-negative On this augmented matrix, if Collies play X Rovers expect $6p_1 + 5p_2$; if Collies play Y Rovers expect $3p_1 + p_2 + 5p_3$; and if Collies play Z Rovers expect $7p_1 + 3p_2 + 4p_3$ We want to maximise M where M only differs by a constant from m and, for each value of p , m is the minimum expected value.	B1 B1 B1	'Add 4', or new matrix written out or equivalent Relating to columns X , Y and Z respectively. Note: expressions are given in the question. For each value of p we look at the minimum output, then we maximise these minima.	[3]																																		
(viii)	$p_3 = \frac{3}{8}$ $M = -\frac{1}{4}$	B1 B1	cao cao	[2]																																		
Total = 19																																						

4 (i)	$8+0+6+5+4$ $= 23$ gallons per minute	M1 A1	$8+0+6+5+4$ or 23 23 with units	[2]
(ii)	At most 6 gallons per minute can enter A so there cannot be 7 gallons per minute leaving it At most 7 gallons per minute can leave F so there cannot be 10 gallons per minute entering it.	B1 B1	Maximum into $A = 6$ Maximum out of $F = 7$	[2]
(iii)	A diagram showing a flow with 12 through E Flow is feasible (upper capacities not exceeded) Nothing flows through A and D Maximum flow through $E = 12$ gallons per minute	M1 M1 A1 B1	Assume that blanks mean 0 12	[4]
(iv) a	If flows through A but not D its route must be $S - A - C - E$, but the flow through E is already a maximum	B1	A correct explanation	[1]
b	$S - (B) - C - D - F - T$ 1 gallon per minute	M1 A1	Follow through their part (iii) 1	[2]
(v)	Flow = $12 + 1 = 13$ gallons per minute Cut through ET and FT or $\{S,A,B,C,D,E,F\}, \{T\}$ $= 13$ gallons per minute Every cut forms a restriction Every cut \geq every flow \square min cut \geq max flow This cut = this flow so must be min cut and max flow	B1 M1 A1 B1	Identifying this cut in any way Use of max flow – min cut theorem min cut \geq max flow This cut = this flow (or having shown that both are 13)	[4]
(vi)	3 gallons per minute Must flow 6 along ET and 7 along FT . Can send 4 into F from D so only need to send 9 through E	B1 B1 B1	3 A correct explanation	[3]
(vii)	A diagram showing a flow of 13 without using BE Flow is feasible and only sends 9 through E	M1 A1	May imply directions and assume that blanks mean 0	[2]
Total =				20

Grade Thresholds

Advanced GCE Mathematics (3890-2, 7890-2)
June 2009 Examination Series

Unit Threshold Marks

7892		Maximum Mark	A	B	C	D	E	U
4721	Raw	72	58	51	44	38	32	0
	UMS	100	80	70	60	50	40	0
4722	Raw	72	56	49	42	35	28	0
	UMS	100	80	70	60	50	40	0
4723	Raw	72	53	46	39	33	27	0
	UMS	100	80	70	60	50	40	0
4724	Raw	72	53	46	39	33	27	0
	UMS	100	80	70	60	50	40	0
4725	Raw	72	49	43	37	32	27	0
	UMS	100	80	70	60	50	40	0
4726	Raw	72	53	46	40	34	28	0
	UMS	100	80	70	60	50	40	0
4727	Raw	72	55	49	43	38	33	0
	UMS	100	80	70	60	50	40	0
4728	Raw	72	62	52	42	33	24	0
	UMS	100	80	70	60	50	40	0
4729	Raw	72	57	48	39	31	23	0
	UMS	100	80	70	60	50	40	0
4730	Raw	72	61	51	41	32	23	0
	UMS	100	80	70	60	50	40	0
4731	Raw	72	55	46	38	30	22	0
	UMS	100	80	70	60	50	40	0
4732	Raw	72	54	47	40	33	27	0
	UMS	100	80	70	60	50	40	0
4733	Raw	72	57	49	41	33	26	0
	UMS	100	80	70	60	50	40	0
4734	Raw	72	55	48	41	34	27	0
	UMS	100	80	70	60	50	40	0
4735	Raw	72	52	45	38	32	26	0
	UMS	100	80	70	60	50	40	0
4736	Raw	72	57	50	44	38	32	0
	UMS	100	80	70	60	50	40	0
4737	Raw	72	52	46	40	34	29	0
	UMS	100	80	70	60	50	40	0

Specification Aggregation Results

Overall threshold marks in UMS (ie after conversion of raw marks to uniform marks)

	Maximum Mark	A	B	C	D	E	U
3890	300	240	210	180	150	120	0
3891	300	240	210	180	150	120	0
3892	300	240	210	180	150	120	0
7890	600	480	420	360	300	240	0
7891	600	480	420	360	300	240	0
7892	600	480	420	360	300	240	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
3890	37.64	54.75	68.85	80.19	88.46	100	18954
3892	58.92	74.42	85.06	91.87	96.04	100	2560
7890	47.57	68.42	83.78	93.17	98.15	100	11794
7892	60.58	80.66	90.76	95.89	98.72	100	2006

For a description of how UMS marks are calculated see:

http://www.ocr.org.uk/learners/ums_results.html

Statistics are correct at the time of publication.

List of abbreviations

Below is a list of commonly used mark scheme abbreviations. The list is not exhaustive.

AEF	Any equivalent form of answer or result is equally acceptable
AG	Answer given (working leading to the result must be valid)
CAO	Correct answer only
ISW	Ignore subsequent working
MR	Misread
SR	Special ruling
SC	Special case
ART	Allow rounding or truncating
CWO	Correct working only
SOI	Seen or implied
WWW	Without wrong working
Ft or ✓	Follow through (allow the A or B mark for work correctly following on from previous incorrect result.)

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OCR is an exempt Charity



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